A Formal Representation of Smith's Gravitational Theory of Happiness

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0. Introduction

This paper offers an elementary possible formal representation of Adam Smith's gravitational theory of happiness, presented in L. Bréban (2014). It does not aim at obtaining new analytical results but, more simply, at showing the mutual consistency of Smith's various propositions, introduced, for most of them, in the *Theory of Moral Sentiments* (Smith 1759; hereafter, TMS).

Broadly speaking, these propositions lead to conclude that in the long-run, people adapt to changes in circumstances caused by prosperous or adverse events. Although Smith asserts that adverse events depress people's mind much more below their "ordinary state of happiness" than prosperous ones, he views these opposite events as only short term shocks, so that an individual's level of happiness tends towards his "ordinary state of happiness". Smith's definition of happiness, of which components are "tranquility of mind" and "enjoyment", allows linking his short-run with his long-run analysis of the effects of adversity and prosperity. It shows that both components should not be considered at the same level: when adverse or prosperous events affect an individual's happiness, it is through the operation of the "tranquility of mind", which appears as an aptitude to "enjoyment". It also explains why adaptation to a new situation or, which is the same, return to the ordinary level of happiness after a deviation, is due to a return to what Smith calls the "usual state of tranquility of mind" along with a trade-off between the objects of enjoyment which were reachable in the previous situation and those which are reachable in the new one. A consequence of the fact that people adapt to whatever becomes their permanent situation is that since adaptation results in a return to their "usual state of tranquility", they can be equally happy in each of these situations. The distinction between short-run and long-run influence of external events on an individual's

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happiness is related to the one between the "natural point of view" and the "impartial spectator's point of view" over the individual's situation. And the adaptation process is the story of a gradual overcoming of the impartial spectator's point of view over the individual's natural point of view, a story which is completed when the individual returns to his "ordinary state of happiness".

In the *Theory of Moral Sentiments*, the exposition of this gravitational theory comes with various examples which illustrate sometimes one of its aspects, sometimes the whole theory. For instance, the dramatic recalling of the imprisonment of the Count de Lauzun (see TMS, III, 3, §31, p. 150) constitutes a typical case of return to the usual state of tranquility along with a tradeoff between different kinds of pleasures (see L. Bréban, 2014, pp. 10-11). The case of the "man who has lost his leg by a cannon shot" (TMS, III, 3, §26, p. 147) is an evocative illustration of the link established between, in the short run, the influence of adverse events and the natural point of view, on the one hand, and between the influence of the same adverse events, but in the long run, and the impartial spectator point of view, on the other hand (see L. Bréban, 2014, pp. 12-14). The last example aims at explaining that a difference between permanent situations might be associated with a lack of difference between levels of happiness. It concerns a comparison between two extreme, opposite, permanent situations: "the most humble station" (let us say, synthetically "poverty"), on the one hand, and "the most glittering and exalted situation" (let us say, "riches"), on the other hand (TMS, III, 3, §31, p. 150). Although presented through a comparative static approach, this last example provides numerous materials which have helped to build its dynamic correlative in order to provide a closer account of what would happen for an individual going from "the most glittering and exalted situation" to "the most humble station", or *vice versa* (see *infra*, section 5), does not raise any peculiar difficulty.

In this paper, the underlying mechanism of convergence to a final long term position (the "ordinary state of happiness"), which is linked to the role granted to virtue in the *Theory of Moral Sentiments* (see L. Bréban 2014, pp. 18-24) is deliberately left aside, since it does not concern directly the central propositions of Smith's gravitational theory of happiness.

After having introduced the relevant notions for the individual's perceptions of his situation (section 1) and the resulting happiness (section 2), the paper deals with the gravitational process properly speaking (section 3), of which a graphical representation is given (section 4), and which is illustrated by Smith's typical examples from the *Theory of Moral Sentiments* (section 5).

1. Perceiving one's situation: Set of Alternatives and Forces of Conception

Let us denote by X the set of alternatives which might be constituted by any kind of objects of enjoyment, such as:

$$X = \{x : x \in \mathbb{R}_+^n\} \tag{A.1}$$

An important feature of Smith's analysis is that the individual's point of view on his own situation (that is, the elements of X that he conceives as available) depends on the degree at which the impartial spectator's point of view has replaced his initial point of view. Smith considers that during the gravitational process, the "force of conception" of the impartial spectator by the individual changes the force of his conception of the elements of X. This means that the description of an individual's situation includes not only his set of alternatives X defined in [A.1], but also the forces of conception of its elements, $\mu(x)$, which can be normalized in the interval [0, 1], $\mu(x) = 0$ and $\mu(x) = 1$ corresponding respectively to an alternative which is viewed as non-available and to a fully conceived alternative. All the couples $(x, \mu(x))$ or, in other words, the graph of the application $\mu(x)$ is therefore the set M belonging to $[0,1]^X$ defined as:

$$M = \{(x, \mu(x)) : x \in X, \mu(x) \in [0,1]\} \tag{A.2}$$

The context of available alternatives, analogous to the "budget" in standard consumer theory, is the subset S of X whose elements are associated to a strictly positive force of conception,

$$S = \{x : (x, \mu(x)) \in M, \mu(x) > 0\} \tag{A.3}[1]$$

Similarly, the impartial spectator's point of view on the agent's situation is given by an element M_Ω of $[0,1]^X$ in which the force of conception of each alternative is Boolean, that is equal either to 0 or to 1, and to the corresponding context of alternatives Ω:

$$M_\Omega = \{(x, \mu_\Omega(x)) : x \in X, \ \mu_\Omega(x) \in \{0,1\}\} \tag{A.4}$$

$$\Omega = \{x : x \in X, \ \mu_\Omega(x) = 1\} \tag{A.5}$$

[A.2] and [A.3] on the one hand, [A.4] and [A.5] on the other hand, respectively express the individual's point of view on his own situation (or, as an immediate result of a change in this situation, what Smith sometimes named the "natural" point of view of the individual), and the impartial spectator's point of view on the individual's situation. In contrast to the individual's

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[1] A possible interpretation of [A.2] and [A.3] is that M is a fuzzy set defined on X, whose characteristic function is $\mu(x)$, and whose (rough) support is S.

point of view, the force of conception $\mu_\Omega(x)$ is clear-cut enough to suppress any doubt on the fact that such x belongs $(\mu_\Omega(x) = 1)$ or does not belong $(\mu_\Omega(x) = 0)$ to Ω, which is clearly expressed by [A.4].

2. Tranquility, Enjoyment, and Happiness

Smith's idea of tranquility of mind is linked to the forces of conception of the alternatives. The level of tranquility T can be viewed as depending on M:

$$[0,1]^X \to \mathbb{R}_+, \ M \mapsto T = T(M) \tag{A.6}$$

The basic properties of $T(M)$ express the idea that

    i) the level of tranquility is all the more high that the forces of conception are high: $\partial T / \partial \mu(x) > 0$;

    ii) the no-tranquility situation corresponds to the case where no object of enjoyment is conceived as available: $M = \{(x, \mu(x)): \forall x \in X, \mu(x) = 0\} \Leftrightarrow T(M) = 0$.

From the fact that $T(M)$ is non-negative and increasing in each $\mu(x)$, we conclude that when the force of conception $\mu(x)$ of an alternative which already belongs to the context of alternatives S increases, all things being equal, the tranquility T also increases. In the same way, when the force of conception $\mu(x)$ of an alternative which does not belong to S rises above 0, the corresponding x becomes a new element of S, and the tranquility T also increases.

The enjoyment E provided by an alternative x of X is considered as depending not only on this alternative, but also on the level of tranquility $T(M)$:

$$(X, \mathbb{R}_+) \to \mathbb{R}_+, (x, T) \mapsto E = E(x, T) \tag{A.7}$$

The enjoyment function is assumed to be increasing in x and T ($\partial E / \partial x, \partial E / \partial T > 0$) and, like a standard function of utility, quasi-concave. Smith's assumption that there is no enjoyment when there is no tranquility can be represented by the existence of a lower bound to $E(x, T)$ such that $E(x, 0) = 0$.

Happiness is defined as the level of enjoyment that an individual reaches on a context S, given his level of tranquility T:

$$H = \max_{x \in S} E(x, T) \tag{A.8}$$

However, the operation described in [A.8] is not that simple. On the one hand, happiness is determined by the level of tranquility and by the context in which alternatives are available. But, on the other hand, since the context S and the tranquility T both depends on M (see [A.3]

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and [A.6]), the possible variations in S and T cannot be viewed as independent. For instance, an increase in the forces of conception of some x, $\mu(x)$, might increase the level of tranquility T and, through this channel, the enjoyment E and the resulting happiness; but the same increase in $\mu(x)$ might also give birth to the entrance in S of new opportunities of enjoyment and, consequently, of happiness.

At this point, it is possible to introduce a conjecture made by Smith about available contexts of alternatives Ω from the impartial spectator's point of view, which has important consequences on both tranquility and happiness. Let us denote F_Ω a subset of the set of parts of X, so that it is constituted by all possible Ω_i ($i \in I$) which correspond to the kind of context which could be available to the individual, from the impartial spectator's point of view. By analogy to the theory of choice, F_Ω can be called a "domain" of alternatives of enjoyment. It is clear that, following Smith, F_Ω cannot be equal to the set of parts of X, since some parts of X cannot be a proper context of alternatives, from the impartial spectator's point of view. For instance, a context in which it would be possible to enjoy simultaneously the pleasure provided by wealth and the one provided by personal liberty is a part of X, but it cannot be a part of F_Ω:

$$F_\Omega = \{\Omega_i : i \in I\} \subset \mathcal{P}(X) \qquad \text{[A.9]}.$$

Each Ω_i is of course linked to a corresponding M_{Ω_i} according to [A.4]-[A.5], the first denoting the impartial spectator's point of view on the available context of alternatives, and the latter denoting the individual's point of view on these alternatives, when he adopts the impartial spectator's point of view. Now, Smith leads us to assume that all these M_{Ω_i} are such that i) the corresponding levels of tranquility are equal to what he calls the "ordinary" level of tranquility, denoted $\bar{T}$, and ii) the corresponding levels of happiness are also equal to an "ordinary" level of happiness $\bar{H}$:

$$\forall \Omega_i \in F_\Omega, \qquad \text{[A.10]}$$
$$T(M_{\Omega_i}) = \bar{T},$$
$$\max_{x \in \Omega_i} E(x, \bar{T}) = \bar{H}.$$

3. Convergence towards the Ordinary State of Happiness

According to Smith, in the long-run, individuals tend to adapt to changes in their situations caused by prosperous or adverse events. Whereas, in the short run, such changes lead to

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deviations from their ordinary state of happiness, in the long run, they converge again toward this state. The distinction between short-run and long-run influence of external events on an individual's happiness is related to the one between the "natural point of view" and the "impartial spectator's point of view" over the individual's situation. Smith considers that convergence towards the ordinary state of happiness corresponds to the gradual overcoming of the impartial spectator's point of view over an individual's natural point of view on his own situation. Various ways of describing the interaction between the natural and the impartial spectator's point of views might be imagined. But whatever they are, they lead to conclude that for each alternative x, its force of conception at date t, $\mu_t(x)$, is changing at a rate $\dot{\mu}_t(x)$, which obviously depends on moral characteristics of the individual, but which is directed toward the force of conception $\mu_\Omega(x)$ which reflects the impartial spectator's point of view. Formally, this means that the signs of $\dot{\mu}_t(x)$ are such that:

$$x \in \Omega \Rightarrow \text{sgn}\left(\dot{\mu}_t(x)\right) = \text{sgn}\left(1 - \mu_t(x)\right) \tag{A.11}$$

$$x \notin \Omega \Rightarrow \text{sgn}\left(\dot{\mu}_t(x)\right) = \text{sgn}\left(-\mu_t(x)\right)$$

The limit of the process described by [A.11] is clearly that the individual's point of view on his own situation, expressed by M_t and S_t (see [A.2] and [A.3]), tends to match the impartial spectator's point of view M_Ω and Ω (see [A.4] and [A.5]):

$$[\text{A.11}] \Rightarrow M_\Omega = \lim_{t \to \infty} M_t \text{ and } \Omega = \lim_{t \to \infty} S_t \tag{A.12}.$$

Smith's conjecture about the ordinary level of tranquility and the ordinary state of happiness [A.10] also leads to the conclusion that tranquility tends to its ordinary level:

$$\Omega \in F_\Omega \Rightarrow \lim_{t \to \infty} T_t = \bar{T} \tag{A.13}$$

And similarly, for the same reason, happiness tends to its ordinary state:

$$\Omega \in F_\Omega \Rightarrow \lim_{t \to \infty} H_t = \max_{x \in \lim_{t \to \infty} S_t} E\left(x, \lim_{t \to \infty} T_t\right) = \max_{x \in \Omega} E(x, \bar{T}) = \bar{H} \tag{A.14}.$$

4. A Graphical Representation of Tranquility and Happiness

Some features of this gravitational process can be illustrated graphically (Figure 1) using a very simplified assumption. Let us suppose that X is constituted only by two alternatives, so that $X = \{x_a; x_b\}^2$, and that the whole range of the impartial spectator's possible point of

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[2] Of course, x_a and x_b might be viewed as composite alternatives, made with different proportions of the same two goods, x_1 and x_2, so that $x_a = (x_1^+, x_2^-)$ and $x_b = (x_1^-, x_2^+)$.

view on the individual situation is given by a domain $F_\Omega = \{\Omega_a; \Omega_b\}$, where $\Omega_a = \{x_a\}$ and $\Omega_b = \{x_b\}$. By contrast, the domain of alternatives of the individual might be $F = \{\Omega_a; \Omega_b; X\}$. Iso-tranquility curves in the $(\mu(x_a), \mu(x_b))$ space, are drawn in Figure 1. They give the level of tranquility associated to each possible M, and each curve refers to all M that provide a same level of tranquility. Since the function of tranquility $T(M)$ [A.6] is increasing in $\mu(x_a)$ and $\mu(x_b)$, the slopes of the iso-tranquility curves are oriented downwards. This expresses the fact that, to keep the same level of tranquility, an increase of the force of conception of one element of X must be offset by a decrease of the force of conception of the other[3]. Always because $T(M)$ is increasing in $\mu(x_a)$ and $\mu(x_b)$, moving from curves in the left-lower part of the graph, like T_1, to curves in the right-upper part, like T_4, leads to increase the level of tranquility. It should also be noted that, as a result from Smith's conjecture on ordinary tranquility [A.10], since Ω_a and Ω_b both belong to F_Ω, the level of tranquility when $M = M_{\Omega a}$ and when $M = M_{\Omega b}$ is the same and equal to the ordinary level of tranquility $\bar{T}$. So that $M_{\Omega a}$ and $M_{\Omega b}$ both belong to the iso-tranquility curve $\bar{T}$.
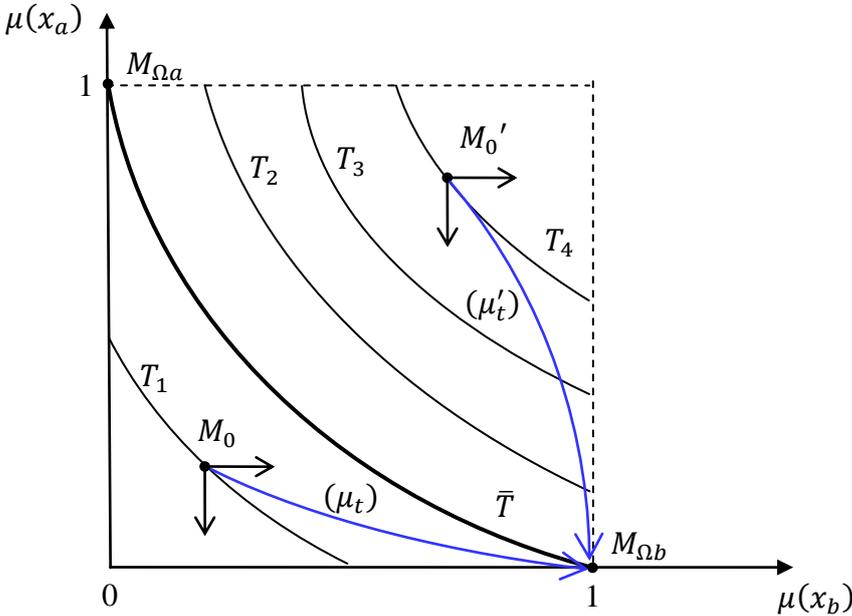

Figure 1 Iso-tranquility curves and convergence toward ordinary tranquility

Now, suppose that the individual initially enjoys an ordinary level of tranquility $\bar{T}$ in $M_{\Omega a}$, where he shares the impartial spectator's point of view on his situation, which means that

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[3] However, there is no textual evidence on the fact that these curves should be convex or concave.

$M = M_{\Omega a}$, so that $S = \Omega_a = \{x_a\}$. Suppose, also, a change in this situation at date $t = 0$, which modifies the alternatives available to the individual from x_a to x_b. From the impartial spectator's point of view, it means that the alternatives and their forces of conception have moved from $M_{\Omega a} = \{(x_a, 1); (x_b, 0)\}$ to $M_{\Omega b} = \{(x_a, 0); (x_b, 1)\}$, at which the same ordinary tranquility might be enjoyed. However, the individual does not adopt immediately such a clear view on his own situation. His assessment of his own new situation, that Smith sometimes call his "natural point of view", might be given, for instance, by M_0 or by $M_0{}'$. In both cases, the force of conception of x_a has decreased, but is not yet equal to zero, and the force of conception of x_b has increased, but is not yet equal to one. The difference between M_0 and $M_0{}'$ is that, in the first case, the individual's short term reaction is tranquility-decreasing, whereas it is tranquility-increasing in the second case. If the individual's change in his point of view over his own situation leads him from $M_{\Omega a}$ to M_0, his lost in the force of conception $\mu(x_a)$ of x_a is not compensated, in terms of tranquility, by the gain in the force of conception $\mu(x_b)$ of x_b, so that he reaches the iso- tranquility curve T_1 where the level of tranquility is superior to the ordinary one. Conversely, if the individual goes from $M_{\Omega a}$ to $M_0{}'$, his lost in $\mu(x_a)$ is more than compensated, in terms of tranquility, by the gain in $\mu(x_b)$, and he reaches the iso- tranquility curve T_4 where the level of tranquility is inferior to the ordinary one[4].

But whatever be the situation at $t = 0$ (M_0 or $M_0{}'$), in the long run, the individual progressively recovers his ordinary level of tranquility through the gradual adoption of the impartial spectator's point of view on his new situation $M_{\Omega b}$. This is achieved through the adjustment process [A.11], which gives rise to a move on the trajectories (μ_t) (from M_0) or (μ_t') (from $M_0{}'$), along which

$$\mu_t(x_a) > 0 \implies \dot{\mu}(x_a) < 0 \text{ and}$$
$$\mu_t(x_b) < 1 \implies \dot{\mu}(x_b) > 0.$$

The result of this process ([A.12] and [A.13]) is that the trajectory (μ_t) or (μ_t') tends to a situation where:

$$\lim_{t \to \infty} M_t = M_{\Omega b} \implies \lim_{t \to \infty} S_t = \Omega_b \text{ and}$$
$$\lim_{t \to \infty} T_t = \bar{T}.$$

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[4] It might as well be assumed that the changes in the forces of conception exactly compensate each other in terms of tranquility, leading to a new M_0 different from both $M_{\Omega a}$ and $M_{\Omega b}$, but lying on the same iso-tranquility curve, so that the initial short term reaction is tranquility-neutral, and $T(M_0) = \bar{T}$.

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At the end of the gravitational process, the individual therefore enjoys, again, his ordinary level of tranquility.

The effect of the gravitational process on happiness and enjoyment is represented in Figure 2, which indicates the level of enjoyment which is provided by each x belonging to X for a given level of tranquility T. The simplification from Figure 1, according to which X was composed of only two elements, is now given up: now, X is composed of any non-negative couple (x_1, x_2):

$$X = \{(x_1, x_2): x_1, x_2 \geq 0\}.$$

Each curve of iso-enjoyment includes all the x which provide a same level e of enjoyment for a given level of tranquility T. In other words, it is the set of solutions x to $E(x, T) = e$. Since the enjoyment function is increasing in x_1 and x_2 and assumed to be quasi-concave, the iso-enjoyment curves are oriented downwards, convex to the origin, and enjoyment rises when moving to a curve located above and on the right of the previous one. Till now, this looks very much like a standard utility representation. But a crucial difference comes from the fact that each family of iso-enjoyment curves in X depends on the value of T. A same point in X therefore belongs to an infinity of iso-enjoyment curves, each of them belonging to a family of curves depending on a different level of tranquility. For instance, the iso-enjoyment curves drawn in black and in blue represent respectively $E(x, \bar{T})$ and $E(x, T')$.
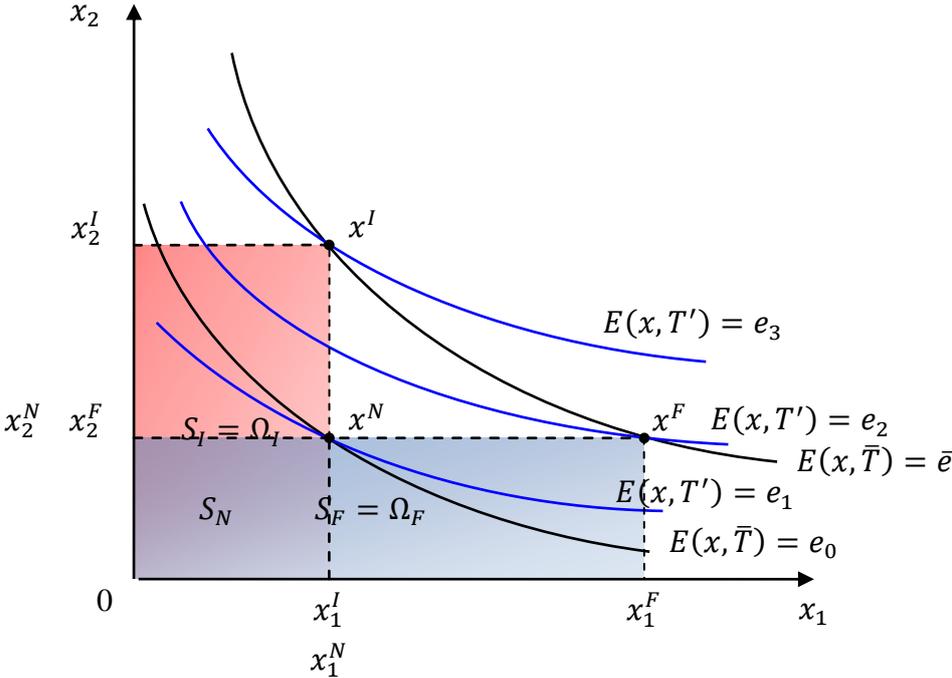

Figure 2 Iso-enjoyment curves and convergence toward ordinary happiness

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Let us now assume (see Figure 2) that the impartial spectator's point of view on the individual's initial situation is $\Omega_I = \{(x_1, x_2): x_1 \in [0, x_1^I], \ x_2 \in [0, x_2^I]\}$. Obviously, this means that for each x in Ω_I, $\mu_{\Omega I}(x)$ is equal to 1, and to 0 otherwise. Let us also suppose that the individual's point of view matches the impartial spectator's point of view, so that his context of alternatives $S_I = \Omega_I$, and that the forces of conceptions of the elements x in X, $\mu_I(x)$ are also equal to $\mu_{\Omega I}(x)$. According to Smith's conjecture [A.10], the individual also enjoys a level of tranquility T_I equal to the ordinary one $\bar{T}$. The iso-enjoyment map of $E(x, \bar{T})$ shows that the maximum of enjoyment assessed over the context of choice Ω_I is provided by $x^I = (x_1^I, x_2^I)$ and is equal to $\bar{e}$. In x^I, always according to Smith's conjecture [A.10], the enjoyment $\bar{e}$ in x^I is equal to the ordinary happiness $\bar{H}$.

Now, suppose that this individual faces a loss in x_2 equal to $(x_2^I - x_2^F)$. From the point of view of the impartial spectator, this loss does not come alone: it is compensated by a gain in x_1 equal to $(x_1^F - x_1^I)$, leading to $\Omega_F = \{(x_1, x_2): x_1 \in [0, x_1^F], \ x_2 \in [0, x_2^F]\}$, with $\mu_{\Omega F}(x)$ is equal to 1 for each x in Ω_F, and to 0 otherwise. But as Smith explains it, such is not the natural short-term point of view of the individual. In several cases (a rich man becoming poor, for instance), his immediate reaction is to focus on the loss of x_2, from which arises a collapse to zero of the forces of conception $\mu_N(x)$ of all $x = (x_1, x_2)$ in Ω_I for which $x_2 > x_2^N$. On the other hand, he even does not imagine that all the alternatives for which x_1 is included between x_1^I and x_1^F are now available, so that the related $\mu_N(x)$ remain equal to zero. The resulting context of his alternatives is therefore $S_N = \{(x_1, x_2): x_1 \in [0, x_1^N], \ x_2 \in [0, x_2^N]\}$.

The consequences in terms of tranquility, enjoyment and happiness are obvious. Since the forces of conception are for each x such that $\mu_N(x)$ is strictly inferior to $\mu_{\Omega I}(x)$ for some x, and equal for the others, the tranquility related to the graph M_N of S_N is $T(M_N) = T'$, which is inferior to the ordinary level of tranquility $\bar{T}$. But since the level of tranquility has changed, the iso-enjoyment map has also changed, and is now related to the enjoyment function $E(x, T')$. The resulting enjoyment on S_N reaches a maximum on $x^N = (x_1^N, x_2^N)$ and amounts to e_1, which is lower than the level of enjoyment e_0 also generated by x^N, but for a higher level of tranquility, $\bar{T}$. Anyways, the individual's happiness, which was at its ordinary state $\bar{H}$ in the initial situation for the alternative x^I, is now $H_N = e_1$, lower than $\bar{H}$.

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This situation is only temporary since in the long run, the individual achieves to adopt the impartial spectator's point of view on his new situation, which results in the equalization of the forces of conception $\mu_F(x)$ to one for all x which belong to Ω_F, and to zero for all other x. The individual's new graph of the forces of conception M_F and new context of alternatives S_F, after his point of view has met the impartial spectator's one, are now $M_F = M_{\Omega F}$ and $S_F = \Omega_F$. We are again under the conditions of Smith's conjecture [A.10] on ordinary tranquility and happiness, so that the individual's level of tranquility is the ordinary one, $T_F = T(M_F) = \bar{T}$. Consequently, his function of enjoyment is, again, $E(x, \bar{T})$, his maximum enjoyment on S_F is reached at $x = x^F$, and is equal to $E(x^F, \bar{T}) = \bar{e}$, which provides, again, the ordinary happiness $\bar{H}$, like in the initial situation, when he was in x^I.

During the gravitational process, both tranquility and happiness have decreased but, while the individual progressively adopts the impartial spectator's point of view, they come closer to their former ordinary level. The details of this process also help understand Smith's not that clear assertion that the difference, if any, between the initial and the final situation, in terms of happiness, is not as important from the impartial spectator's point of view as it is from the individual's. In Figure 2, the first difference is $E(x^F, \bar{T}) - E(x^I, \bar{T}) = 0$, since x^I and x^F are on the same iso-enjoyment curve if $T = \bar{T}$. For the impartial spectator, the individual reaches the same ordinary state of happiness $\bar{H}$ either at x^I or at x^F. On the contrary, from the natural point of view associated with a level of tranquility T' lower than the ordinary one, the difference $E(x^F, T') - E(x^I, T') = e_2 - e_3$ is negative, and the individual overestimate his happiness at x^I, when compared to his happiness at x^F.

5. Typical Illustrations

Smith's comparison between what he calls the "most humble station" and the "most glittering and exalted situation" constitutes a first typical illustration of his gravitational theory of happiness that could be reread through this formal representation (see TMS, III, 3, §31, p. 150). Other examples, like the "the man with a wooden leg" (see TMS, III, 3, §28, p. 148) or "the count of Lauzun" (see TMS, III, 3, §31, p. 150), might also be easily restated as special instances of a move from the best to the worst situation.

The objects of enjoyment, from which happiness is derived, are said to be "almost" the same in the "most humble station" and in the "most glittering and exalted situation", on the double exception of "personal liberty" which is the privilege of the former and of "the frivolous pleasures of vanity and superiority" which are the privilege of the latter. Let us denote these two positions x_p and x_r (p standing for poverty, and r for riches) and, to simplify the discussion, let us suppose that $X = \{x_p; x_r\}$, and that the whole range of the impartial spectator's possible point of view on the individual situation is given by a domain $F_\Omega = \{\Omega_p; \Omega_r\}$, where $\Omega_p = \{x_p\}$ and $\Omega_r = \{x_r\}$ respectively correspond to his point of view on the "most humble situation" and on the "most glittering and exalted situation". By contrast, the domain of alternatives of the individual might be $F = \{\Omega_p; \Omega_r; X\}$. Two cases could be derived from this comparison:

i.   The case of an individual who goes from the "most glittering and exalted situation" to the "most humble station";

ii. The case of an individual who goes from the "most humble station" to the "most glittering and exalted situation".

These two cases will be addressed successively.

5.1. From the "most glittering and exalted situation" to the "most humble station"
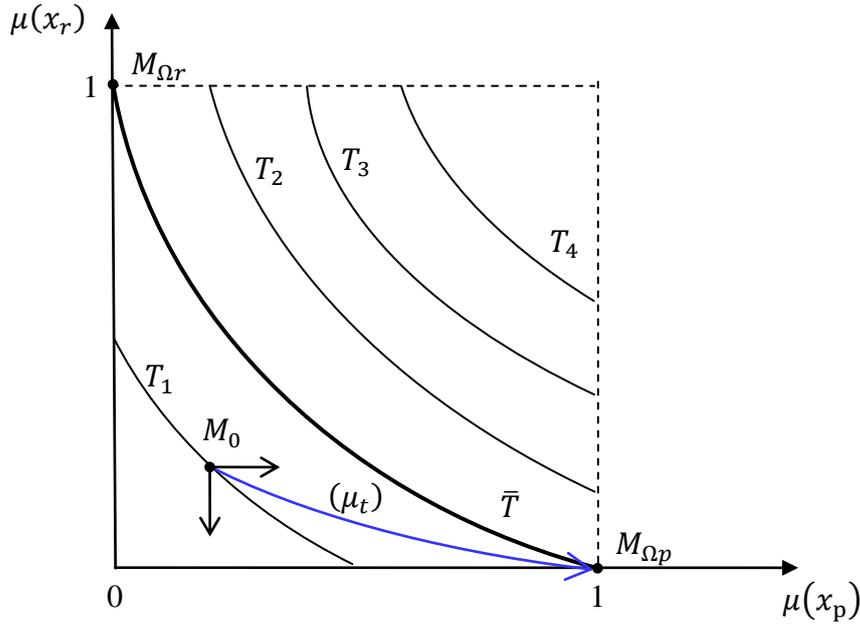
Figure 3 Tranquility from the "most glittering and exalted situation" to the "most humble station"

Suppose (see Figure 3) that the individual is initially in the "most glittering and exalted situation" and shares the impartial spectator's point of view on his situation, so that $M = M_{\Omega r}$ and $S = \Omega_r = \{x_r\}$. As a result, in his initial situation, he enjoys an ordinary level of tranquility $\bar{T}$ in $M_{\Omega r}$. Suppose, also, a change in this situation at date $t = 0$ which leads him from this "most glittering and exalted situation" to the "most humble station". This change modifies the alternatives available to the individual from x_r to x_p. According to the impartial spectator, in this situation, the absence of "the frivolous pleasures of vanity and superiority" is compensated by the pleasure of "personal liberty", so that, his point of view moves from $\Omega_r = \{x_r\}$ to $\Omega_p = \{x_p\}$. However, the individual does not adopt immediately such a clear view on his own situation. This change of situation makes him think that besides the absence of enjoyment of "personal liberty", he can't enjoy "the frivolous pleasures of vanity and superiority". His assessment of this new situation is given by M_0: his force of conception of x_r has decreased, but is not yet equal to zero, and his force of conception of x_p has increased, but is not yet equal to one. The individual's short term reaction is tranquility-decreasing, so that, he reaches the iso- tranquility curve T_1 where the level of tranquility is inferior to the ordinary one.

But in the long run, the individual progressively recovers his ordinary level of tranquility through the gradual adoption of the impartial spectator's point of view on his new situation $M_{\Omega p} = \{(x_r, 0); (x_p, 1)\}$, that is, that the "most humble station" allows the enjoyment of

"personal liberty". The adjustment process [A.11] gives rise to a move on the trajectory (μ_t) (from M_0) along which

$$\mu_t(x_r) > 0 \Longrightarrow \dot{\mu}(x_r) < 0 \text{ and}$$

$$\mu_t(x_p) < 1 \Longrightarrow \dot{\mu}(x_p) > 0.$$

The result of this process ([A.12] and [A.13]) is that the trajectory (μ_t) tends to a situation where:

$$\lim_{t\to\infty} M_t = M_{\Omega p} \Longrightarrow \lim_{t\to\infty} S_t = \Omega_p \text{ and}$$

$$\lim_{t\to\infty} T_t = \bar{T}.$$

At the end of the gravitational process, the individual therefore realizes that the "most humble station" is not inconsistent with the enjoyment of his ordinary level of tranquility.

With regard to the effect of this change of situation on enjoyment, let us give up the idea that there are only two alternatives of enjoyment x_p and x_r. It will be now assumed that X is composed of any non-negative couple (x_l, x_f) such that

$$X = \{(x_l, x_f): x_l, x_f \geq 0\}$$

(l and f respectively standing for 'personal *liberty*' and '*frivolous* pleasures').

Like before, we suppose that the individual's point of view on his initial situation matches the impartial spectator's one, so that his context of alternatives is $S_r = \Omega_r$, and the forces of conceptions of the elements x in X, $\mu_r(x)$ are also equal to $\mu_{\Omega r}(x)$. According to [A.10], the individual also enjoys a level of tranquility T_r equal to the ordinary one $\bar{T}$. The iso-enjoyment map of $E(x, \bar{T})$ shows that the maximum of enjoyment assessed over the context of choice Ω_r is provided by $x^r = (x_l^r, x_f^r)$ and is equal to $\bar{e}$ (see figure 4). In x^r, always according to [A.10], the enjoyment $\bar{e}$ is equal to the ordinary level of happiness $\bar{H}$.

Now, suppose that this individual faces a loss in x_f equal to $(x_f^r - x_f^p)$ where $x_f^r > x_f^p$. From the point of view of the impartial spectator, this loss is compensated by a gain in x_l equal to $(x_l^p - x_l^r)$ where $x_l^p > x_l^r$, leading to $\Omega_p = \{(x_l, x_f): x_l \in [0, x_l^p], \ x_f \in [0, x_f^p]\}$, with $\mu_{\Omega p}(x)$ equal to 1 for each x in Ω_p, and to 0 otherwise. But the individual's natural short-term point of view leads him to focus on the loss of x_f, from which arises a collapse to zero of the forces of conception $\mu_N(x)$ of all $x = (x_l, x_f)$ in Ω_r for which $x_f > x_f^N$. On the other hand, he even does not imagine that all the alternatives for which x_l is included between x_l^r and x_l^p are now available, so that the related $\mu_N(x)$ remain equal to zero. The resulting context of his alternatives is therefore $S_N = \{(x_l, x_f): x_l \in [0, x_l^N], \ x_f \in [0, x_f^N]\}$.

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Since the forces of conception are for each x such that $\mu_N(x)$ is strictly inferior to $\mu_{\Omega r}(x)$ for some x, and equal for the others, the tranquility related to the graph M_N of S_N is $T(M_N) = T'$, which is inferior to the ordinary level of tranquility $\bar{T}$. And the iso-enjoyment map is now related to the enjoyment function $E(x, T')$. The resulting enjoyment on S_N reaches a maximum on $x^N = \left(x_l^N, x_f^N\right)$ and the individual's happiness, which was at its ordinary state $\bar{H}$ in the initial situation for the alternative x^r, is now $H_N = e_1$, lower than $\bar{H}$.

But in the long run, the individual achieves to adopt the impartial spectator's point of view on his new situation, which results in the equalization of the forces of conception $\mu_p(x)$ to one for all x which belong to Ω_p, and to zero for all other x. The individual's new graph of the forces of conception M_p and new context of alternatives S_p are now $M_p = M_{\Omega p}$ and $S_p = \Omega_p$. According to the conjecture [A.10], the individual's level of tranquility is the ordinary one, $T_p = T(M_p) = \bar{T}$. Consequently, his function of enjoyment is, again, $E(x, \bar{T})$, his maximum enjoyment on S_p is reached at $x = x^p$, and is equal to $E(x^p, \bar{T}) = \bar{e}$, which provides, again, the ordinary happiness $\bar{H}$. This allows to conclude that, according to Smith, people can be equally happy in the "most glittering and exalted situation" and in the "most humble station".
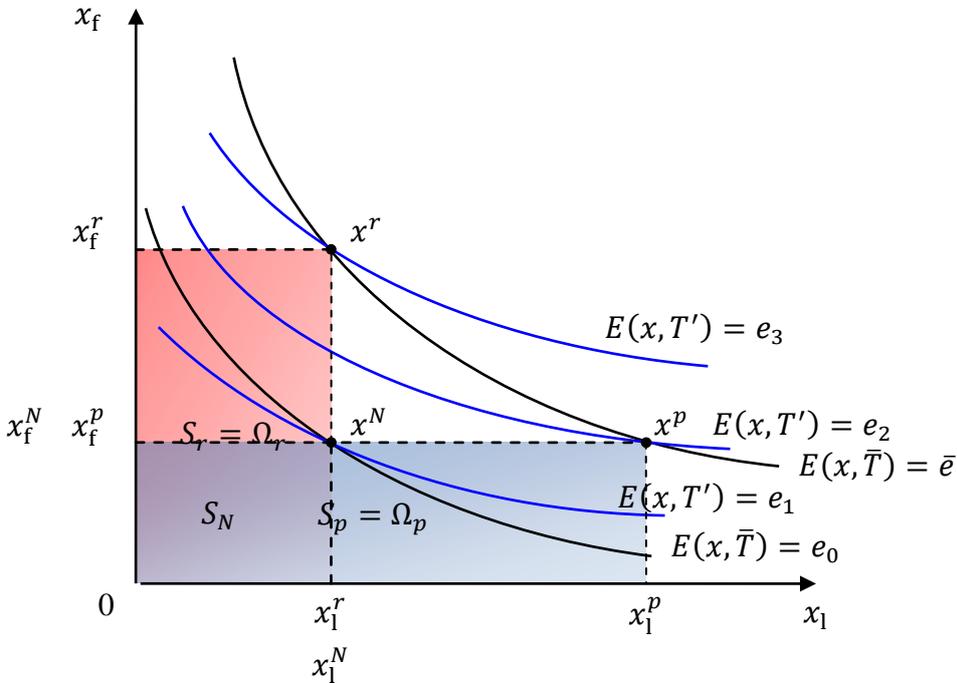

Figure 4 Enjoyment from the "most glittering and exalted situation" to the "most humble station"

5.2. From the "most humble station" to the "most glittering and exalted situation"
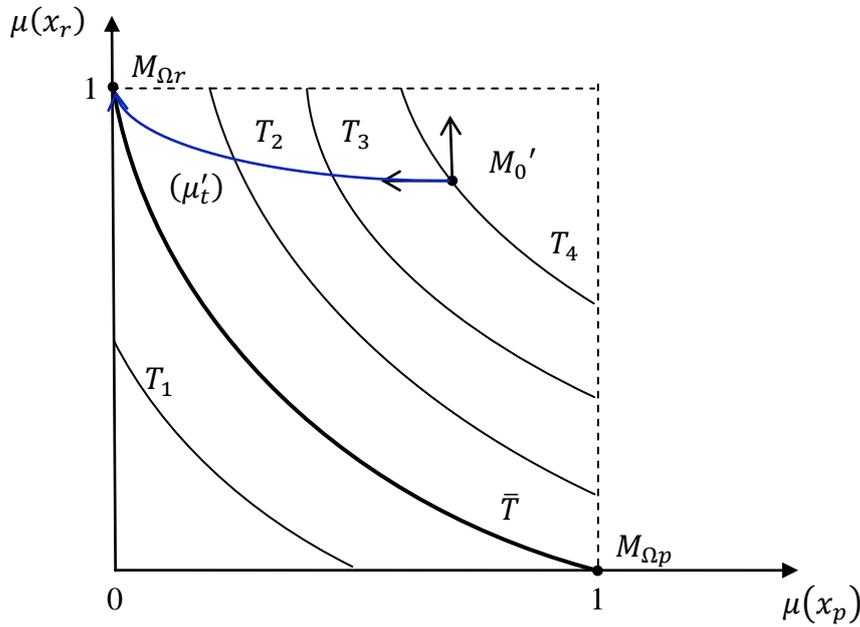

Figure 5 Tranquility from the "most humble station" to the "most glittering and exalted situation"

By contrast to the previous case, suppose this time that the individual is initially in the "most humble station". As before, in his initial situation, he shares the impartial spectator's point of view on his situation, so that, $M = M_{\Omega p}$ and $S = \Omega_p = \{x_p\}$ and thus, enjoys an ordinary level of tranquility $\bar{T}$. Suppose, also, a change in this situation at date $t = 0$, which leads him from the "most humble station" to the "most glittering and exalted situation". This time, the change modifies the alternatives available to the individual from x_p to x_r. According to the impartial spectator, in this situation, the gain of "the frivolous pleasures of vanity and superiority" comes with a loss of the pleasure of "personal liberty", so that, his point of view moves from $\Omega_p = \{x_p\}$ to $\Omega_r = \{x_r\}$. However, as in the previous case, the individual does not immediately view his situation from this angle. In a first time, he thinks that besides the enjoyment of "personal liberty", he can enjoy "the frivolous pleasures of vanity and superiority". His assessment of this new situation is given by M_0': his force of conception of x_r has increased, but is not yet equal to one, and his force of conception of x_p has decreased, but is not yet equal to zero. Contrary to the opposite change of situation, the individual's short term reaction is now tranquility-increasing, so that, he reaches the iso- tranquility curve T_4 where the level of tranquility is superior to the ordinary one.

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But, like in the former case, in the long run, the individual progressively recovers his ordinary level of tranquility through the gradual adoption of the impartial spectator's point of view on his new situation $M_{\Omega r}$. And this gradual acceptance leads him to realize that the "most glittering and exalted situation" does not allow the enjoyment of "personal liberty". The adjustment process [A.11] gives rise to a move on the trajectories (μ'_t) (from M_0') along which

$$\mu_t(x_p) > 0 \Longrightarrow \dot{\mu}(x_p) < 0 \text{ and}$$
$$\mu_t(x_r) < 1 \Longrightarrow \dot{\mu}(x_r) > 0.$$

The result of this process ([A.12] and [A.13]) is that the trajectory (μ'_t) tends to a situation where:

$$\lim_{t\to\infty} M_t = M_{\Omega r} \Longrightarrow \lim_{t\to\infty} S_t = \Omega_r \text{ and}$$
$$\lim_{t\to\infty} T_t = \bar{T}.$$

At the end of the gravitational process, the individual therefore realizes that the "most glittering and exalted situation" leads to the same ordinary level of tranquility as the "most humble station".

As for the effect of this change of situation on enjoyment, we will, like in the previous case, give up the idea that there are only two alternatives of enjoyment, so that $X = \{(x_l, x_f): x_l, x_f \geq 0\}$. We also assume now that the individual's initial context of alternatives is $S_p = \Omega_p$, that the forces of conceptions of the elements x in X, $\mu_p(x)$ are also equal to $\mu_{\Omega p}(x)$, so that, he enjoys a level of tranquility T_p equal to the ordinary one $\bar{T}$. The maximum of enjoyment assessed over the context of choice Ω_p is provided by $x^p = (x_l^p, x_f^p)$ and is equal to $\bar{e}$ (see figure 6), itself equal to the ordinary level of happiness $\bar{H}$.

Now, suppose that this individual faces a gain in x_f equal to $(x_f^r - x_f^p)$ where $x_f^r > x_f^p$. From the point of view of the impartial spectator, this gain is compensated by a loss in x_l equal to $(x_l^p - x_l^r)$ where $x_l^p > x_l^r$, leading to $\Omega_r = \{(x_l, x_f): x_l \in [0, x_l^p], x_f \in [0, x_f^p]\}$, with $\mu_{\Omega r}(x)$ equal to 1 for each x in Ω_r, and to 0 otherwise. But the individual's natural short-term point of view leads him to focus on the gain of x_f, from which arises an increase of the forces of conception $\mu_N(x)$ of all $x = (x_l, x_f)$ out of Ω_p for which $x_f < x_f^N$. On the other hand, he even does not imagine that all the alternatives for which x_l is included between x_l^r and x_l^p are now unavailable, so that the related $\mu_N(x)$ remain superior to zero. The resulting context of his alternatives is therefore $S_N = \{(x_l, x_f): x_l \in [0, x_l^N], x_f \in [0, x_f^N]\}$.

Since the forces of conception are for each x such that $\mu_N(x)$ is strictly superior to $\mu_{\Omega_p}(x)$ for some x, and equal for the others, the tranquility related to the graph M_N of S_N is $T(M_N) = T'$, which is superior to the ordinary level of tranquility $\bar{T}$. And the iso-enjoyment map is now related to the enjoyment function $E(x, T')$. The resulting enjoyment on S_N reaches a maximum on $x^N = (x_l^N, x_f^N)$ and the individual's happiness, which was at its ordinary state $\bar{H}$ in the initial situation for the alternative x^p, is now $H_N = e_1$, higher than $\bar{H}$.

Again, in the long run, the individual achieves to adopt the impartial spectator's point of view on his new situation, which results in the equalization of the forces of conception $\mu_p(x)$ to zero for all x which does not belong to Ω_r, and to one for all other x which belong to Ω_r. The individual's new graph of the forces of conception M_r and new context of alternatives S_r are now $M_r = M_{\Omega_r}$ and $S_r = \Omega_r$. According to the conjecture [A.10], the individual's level of tranquility is the ordinary one, $T_r = T(M_r) = \bar{T}$. Consequently, his function of enjoyment is, again, $E(x, \bar{T})$, his maximum enjoyment on S_r is reached at $x = x^r$, and is equal to $E(x^r, \bar{T}) = \bar{e}$, which provides, again, the ordinary happiness $\bar{H}$.

This means that a move from the "most humble station" to the "most glittering and exalted situation" by means of a prosperous event, corresponds to a trade off where the pleasure of personal liberty is exchanged for the frivolous pleasures of vanity and superiority, the long-run level of happiness remaining constant.
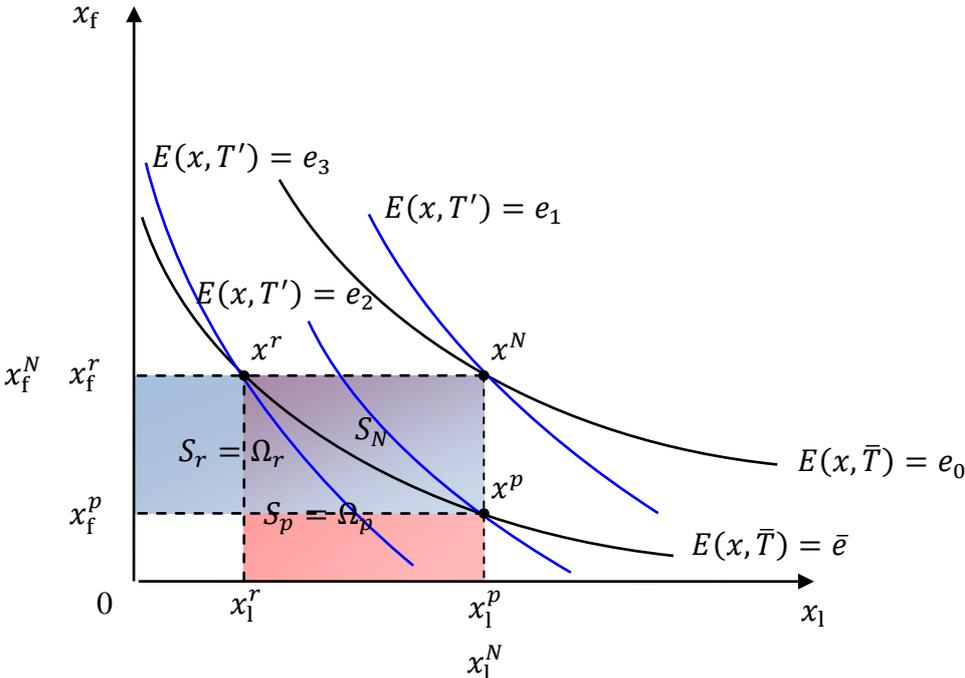

Figure 6 Enjoyment from the "most humble station" to the "most glittering and exalted situation"

6. Concluding Remarks

On top of showing the mutual consistency of Smith's various propositions introduced in the *Theory of Moral Sentiments*, the previous representation allows distinguishing the author's analysis of happiness from a standard approach, closer to a more familiar hedonistic and welfarist approach. Though, at first sight, both appear similar, they differ on several important points.

i. Both approaches share an identical starting point, that is, the identification of a set of alternatives X which could be composed of any type of objects which might contribute to satisfaction or, in Smith's words, enjoyment. However, something specific is added, within a Smithian approach to this set of alternatives: the forces of conception $\mu(x)$ of its elements. This means that from a Smithian point of view, the initial question is not only that of the mere existence of objects, but also of the strength of their existence for the agent.

ii. Within both approaches, we can define a relevant domain of alternatives F which consists in a subset of non empty parts of X, and contexts of alternatives S which are the elements of F. This last constitutes what Smith called the "natural point of view" of the individual. However, he adds again something more to such an initial natural point of view: the impartial spectator's point of view. This last one gives rise to a different domain of alternatives F_Ω, which acknowledges that all parts of X are not potentially reachable by the agent.

iii. Whereas in a standard approach, the satisfaction derived from an element of X only depends on this element, in the Smithian approach it also depends on the level of tranquility of mind which is, itself, linked to the forces of conception. As a result, a same alternative x might give rise to an infinity of levels of enjoyment (and thus belongs to an infinity of iso-enjoyment curves) depending on the level of tranquility. An immediate consequence is that variations in the tranquility of mind might change the agent's preferences over the elements of X.

iv. An important feature of Smith's analysis, which has no genuine equivalent in a standard approach, is that from the impartial spectator's point of view, the selective domain of choice of alternatives F is such that the same "ordinary level of happiness", associated with a same "ordinary level of tranquility" might be obtained from any element of F_Ω.

v. The last feature which has no equivalent in standard approach is the existence of a dynamics which concerns the forces of conception and, as a result, the domain of choice, tranquility, enjoyment and happiness. This dynamics is assumed by Smith to lead more or less quickly to the adoption by the individual of the impartial spectator's point of view. So that in spite of the various positive or negative shocks which he encounters during his life, he always tends to recover his ordinary level of happiness. Although, after such a shock, his natural point of view leads him to prefer such or such permanent situation, the adoption of the impartial spectator's point of view brings him to conclude that all permanent situations might provide the same ordinary level of happiness.

7. References

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