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► To cite this version:

Jean-Marc Bottazzi, Mário Páscoa, Guillermo Ramírez. Do Security Prices Rise or Fall When Margins Are Raised?. 2017. hal-01648215

HAL Id: hal-01648215

<https://paris1.hal.science/hal-01648215>

Preprint submitted on 25 Nov 2017

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DO SECURITY PRICES RISE OR FALL WHEN MARGINS ARE RAISED?

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Abstract

When repo margins are raised by a central clearing counterparty (CCP), the impact on security prices depends on whether the market is ‘*long*’ or ‘*short*’. As both the long and the short must post margins, the price impact depends on which side is more leveraged: traders long in the security or those short-selling it. If a raised margin forces more position unwind from the long than from the short, the price will go down to clear the market. However, if short positions are more hit than the long ones, a raised haircut leads to a higher security price!

¹ We are thankful to comments from the audience at the SAET 2017 conference (June 2017). G. Ramírez was supported by a doctoral grant given by FCT, Portugal (grant BD 74704/2010). Páscoa and Ramírez acknowledge support from project PTDC/IIM-ECO/5360/2012 (FCT, Portugal).

1 Introduction

Without rehypothecation, raised margin typically decreases the price of the collateral (houses prices for mortgage, or the price of other collateral used to asset back securities). In fact, the only leverage provided is to the long side of the collateral market, who is funding collateral purchases. Reduced leverage, everything else being equal, means less buying and hence a lower collateral price.

The situation is less clear, however, in the presence of rehypothecation, for example in the securities markets. Securities serve as collateral when financing their own purchase and then cash lenders manage to repledge or short sell² the collateral they have accepted. That is, in these markets, self-collateralization combined with collateral reuse allow for short positions. Since leverage is provided to both the long and the short side, the price impact of a leverage reduction can also go either way. **With counterparty clearing houses (CCPs)**, both the long side and the short side need to post margin and are thus affected by variation of haircut³: it is well documented how higher volatility (or just higher spreads on yields of sovereign bonds relative to a AAA benchmark) make CCPs (and futures exchanges) increase the initial margins charged in repo trades to both sides of the market⁴, agents that pledge securities and agents that accept these as collateral for cash loans. positions

It has been argued, in most of the literature we review below, that the impact of higher haircuts is always to lower the price of the collateral. If this were true this would mean that raising margin is always procyclical : if the price of the collateral

² The counterparties to these shorts can also repledge the securities (an indirect rehypothecation) and a new iteration starts.

³ Stricly speaking, the haircut indicates, in percentage terms, how the loan falls below the collateral value, whereas the initial margin expresses the collateral as a percentage of the loan (it is therefore the inverse of one minus the haircut).

⁴ It should be clear that a similar analysis can be done replacing security with deliverable futures and haircut with futures initial margin.

is already under pressure, a higher haircut pushes it further down in some vicious cycle that in the worst case can feed on itself.

However, to take just an example, if we observe what happened in the European sovereign debt crisis, we see yields dropping after some margins were raised. This is more notorious for the 10 year government bonds of the following countries and for the following episodes: Ireland, 12 and 26 November 2010 and again for the six consecutive margin increases from April through June 2011; Italy, 9 Nov 2011 and July 2012; Portugal, 11 May 2011 and 29 June 2011; Spain, 20 June 2012.

There are nevertheless also some down spirals in prices⁵, but this is certainly not the most frequent market reaction after margins are raised. Later, in particular for Irish and Spanish bonds, as the crisis started to fade away, speculation decreased and the fall in haircuts was accompanied by a fall in yields.

There may be other factors behind such yield movement, but it does suggest that the impact of raising margin is *not always* to push the price of the collateral down. Our view is that *in securities markets* it depends on *positioning*. For this reason, as positioning changes quickly over time, one can expect the impact of margins to be more visible in short term market reactions rather than in long **term trends**. It is well-known that at various times speculative positions were quite significant. This raises the possibility that raising haircut could at times have had more impact on the short side. And a contributing factor of yields going down is shorts being the leveraged traders being forced to deleverage when margin is raised. In market parlance, a security market is *long* when the longs are **leveraged and short when it is the shorts that are more leveraged instead**. We will give a more precise definition for a terminology that superficially seems to clash with market clearing (which requires longs and shorts to be matched). The emphasis is on where the leverage is when the haircut rises. If the short side gets to be

⁵ 19 November 2010 and 25 March 2011 for Ireland; throughout April 2011 and on 14 June 2011 for Portugal

reduced more than the long side (which happens when ‘the market is short’), then the price of the security going up, everything else being equal, enables markets to clear.

2 Relation with the Literature

Most of the literature emphasized the procyclical role of repo haircuts and the resulting propagating down-spiral. Adrian and Shin (2009) observe a strong positive correlation between leverage and balance sheet size of US financial intermediaries, inferring from it a procyclical role of leverage. A theoretical work by Brunnermeier and Pedersen (2009) notices that short selling also requires capital in the form of margin and provides an interesting result pointing out that a margins spiral arises when long margins are decreasing in prices or short margins are increasing in prices. It seems to be implicit that prices might increase with short margins and that a counter-cyclical pattern could happen if the short margins were decreasing in prices, just like the long margins.

The observed, not so infrequent, escapes of what just before looked like an implacable vicious circle logic motivated us. When, in response to a price drop and volatility, margins are increased and the price subsequently rises, is the raised margin a contributing factor to this rise or does the rise in price happen in spite of a headwind margin effect? We suggest that the margin effect can sometimes be supportive: *higher margin also can nudge prices higher*.

In the empirical literature, the analysis that Gorton and Metrick (2012) did of the 2008 financial crisis argued that the spiral interaction of repo haircuts and prices of structured securities led to a run on repo and insolvency of the US banking system. While that pro-cyclicality may have actually occurred in what was then a long market for those securities, Krishnamurthy et al. (2014) downplayed its

magnitude and impact on the whole financial system. These different assessments may be due, as Infante (2015) pointed out, to Gorton and Metrick (2012) having focused on bilateral trades, while in the large US tri-party market haircuts may have been changed much less. We argue for a balanced view: in a market with both longs and shorts, *leverage by itself does not have to be procyclical depending where leverage is more prevalent*.

In any case, the presumed procyclical nature of margins set by CCPs (more systematic than in over-the-counter (OTC) market) has even led to the view that CCPs might be factor of instability. This view was nevertheless met with some scepticism by European policy-makers that value CCPs' role as a firewall against the propagation of default shocks and find an increasingly centrally cleared market easier to monitor (see Constâncio (2012)).

In the case of the European sovereign debt crisis, a lot of positioning was done by banks (hedges of illiquid position) and hedge funds. We find tracking CCP margin a good way to gauge evolution of margins relevant to those agents. Not just because Banks tend to use CCPs but all relevant margins have been moving broadly together: (1) hedge funds use prime brokers who act as clearers for them (also charging margin regardless of whether the trader is short or long) (2) Futures contracts (where the exchange asks for initial margin regardless of position direction). Our focus on central clearing was not chosen only for analytical purposes but also seems to fit better with what happened in that crisis we chose to illustrate our point.

There is an important literature that has studied collateral in the general equilibrium (GE) theory tradition and this is the literature that is closest related to our work. The papers by Geanakoplos (1997) and Geanakoplos and Zame (1997) set the stage: radically changing the way borrowing is treated in GE models, replacing abstract negative asset positions (subject to exogenous naive constraints) by loans

secured by assets. Subsequently, many contributions explored implications of how collateral margins are set (like absence of Ponzi schemes⁶, welfare properties⁷ or the volatility of collateral prices⁸) but the most relevant to our theme are the papers on how leverage impacts asset prices, stemming from Geanakoplos (2003). In particular, the papers by Geanakoplos (2010), Fostel and Geanakoplos (2012) and Fostel and Geanakoplos (2015), contemplate a continuum of traders, with different beliefs on binominal asset returns, and compare unsecured borrowing with borrowing against a collateral which is either a durable good or a financial asset that cannot be short-sold.

One of the main insights of these three seminal papers is that leverage makes the price of the asset that serves as collateral rise. To start, leverage equilibrium prices were shown to be greater than no-leverage prices. It was noticed that prices increase with exogenous loan-to-value ratios, but the emphasis was then placed on the endogenous determination of leverage (ruling out default on non-recourse loans). The endogenous deleverage always reinforced a fall in prices caused by bad news about the worse case payoff.

We use Fostel and Geanakoplos' (2012) and (2015) binomial model with a continuum of degrees of agents' optimism on asset payoffs. We find it surprisingly versatile and apt to illustrate rich and subtle phenomena of general relevance. We use it to show that lower margin might not always raise asset prices, if *assets can be short sold*. The key difference - more important than recourse - is if the collateral may be reused by the creditor on whose hands it was pledged. In a nutshell: Asset prices increase with leverage in a *long* market and decrease in a *short* market. In a long market the optimists are leveraged, while it is the pessimists who are if it is *short*. To make the comparison easier, our examples are as close as possible to the

⁶ See Araujo, Páscua and Torres-Martinez (2002) and Kubler and Schmeders (2003).

⁷ See Araujo, Kubler and Schommer (2012) and Geanakoplos and Zame (2014).

⁸ See Brumm, Grill, Kubler and Schmeders (2015).

framework used in those papers, in particular to the benchmark model in Fostel and Geanakoplos (2012).

As in Fostel and Geanakoplos (2012), we contemplate a binomial economy (the initial node is followed by two nodes) and two assets, one that can be pledged as collateral and another one that cannot. The former has stochastic returns and the latter is riskless. We can interpret the former as a risky security and the latter as cash. Also as in Fostel and Geanakoplos (2012) and Fostel and Geanakoplos (2015), there is a continuum of agents, with the same initial holdings of assets at the first date but differing in the weights attached to the two states of nature in their linear utility functions.

We depart from Fostel and Geanakoplos (2012) and Fostel and Geanakoplos (2015) in three aspects. First and foremost the collateral can be reused (and short sold). Second, loans are recourse, as is typically the case in repo (with an exception allowed by the Fed during the 2008 crisis). Default is a bankruptcy trigger⁹ and, therefore, not so common in repo markets. For this reason, and also for simplicity and tractability, we focus on asset price fluctuations and look for full repayment outcomes¹⁰. Third, agents have endowments (of the riskless asset) also at the two nodes of the second date. Such endowments played no role in ensuring solvency in the non-recourse case, but we can now choose them high enough so that the shorts can contemplate maximum leverage for a given level of haircut, without going bankrupt at the second date.

Our model is used to construct two benchmark cases for a repo market cleared through a CCP. One where the market is short and raising haircuts/initial margin

⁹ The collateral value can be below the promised repurchase price and yet default might not occur. On the other hand, repo collateral is exempt by the US bankruptcy code from automatic stay and the cash lender can liquidate the collateral immediately, but if the liquidation value exceeds (or is below) the repurchase price, the difference is deemed property of the insolvent estate (or can be claimed against that estate, respectively).

¹⁰ In Bottazzi, Páscua and Ramirez (2017), we work out how bankruptcy can be modelled but here we leave that intricate non-convex problem aside.

leads to higher security prices (as in the sovereign debts episodes mentioned above). Another, where the market is long and raising the haircut makes that the security price go down. What determines whether the market will be long or short is how security returns vary across nodes by comparison with endowments of the riskless asset. The latter might be too low (in the state where the security has higher return) to allow for the shorts to lever up to what the haircut allows for.

The paper shows how with rehypothecation one can build counter-examples to well-known results in the absence of such re-use. In the next section we provide a very brief motivation on the basis of observed haircut. In section 4 we describe the model of centrally cleared repo and in the following sections we present the examples, discussing in more detail a short market where prices rise in response to higher margin. That being said we cannot claim that haircuts have a stabilizing effect by themselves. First there are many markets where leverage is only offered to the long side. Also when shorts are possible, leveraged positioning could stabilise the price when margin is raised only if the market would be well equipped, in the sense of having plenty of leveraged shorts willing to take profit in any excessive down price move.

3 Suggestive episodes

Let us now look into our illustrative example. We chose this example because of easy access to data: the way CCPs raise repo margins when security prices fall has been well documented and elaborated on in the literature. For European government bonds, the major clearing houses are LCH.Clearnet Ltd (London), LCH.Clearnet SA (Paris) and Cassa di Compensazione e Garanzia (CC&G) SpA (Rome). The first one tends to raise initial margin when spreads exceed 450 basis points over a 10 year AAA benchmark. The second and third use the same

margins model based on several indicators. Italian government bonds are the largest collateral pool in European repo and are cleared by both LCH.Clearnet SA and CC&G. Irish and Portuguese bonds (together with many bonds issued by less risky countries) are traded by CLH.Clearnet Ltd, while Spanish bonds may be cleared by both LCH branches. Several papers did a very detailed analysis of how initial margins on 10 year bonds issued by these 4 governments responded to yields changes¹¹. We are interested in the other direction (but more short term) ...and use these same markets as illustration.

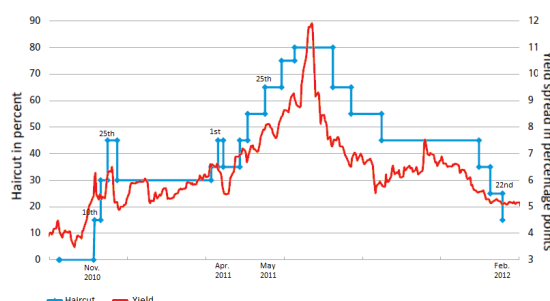


Figure 1: Haircuts and yield spread for 10 year Irish government bonds on 10 year German Government bond. Source: Armakola, Douady, Laurent and Molteni (2016)

We notice that there are several episodes where margins went up but yields shortly fell. For the 10 year *Irish bond*, figure 1 shows for late 2010 and the first half of 2011 several instances when haircut rises were immediately followed by falls in yields. For example, when in response to a yields increase (by 9% on the 9th of November 2010), LCH.Clearnet Ltd. announced on the 10th of November that the initial margin would be raised on the 12th to 15%. This was followed by decreases in yields of 7.9% and 2.7% on the 12th and the next market date, respectively. On the 1st of April 2011, initial margins were raised to 45% and yields dropped that

¹¹ We borrow Figure 1 on the Irish bond from Altenhofen and Lohff (2013) and Figures 2 and 3 on the other three bonds from Armakola, Douady, Laurent and Molteni (2016). In the Supplementary Material we present tables reporting haircuts and yields for the most relevant dates.

day (by 2.62%), the following day (by 2.32%) and very significantly by 5.9% on the third subsequent market day (the 6th, followed by smaller reductions in the next four days).

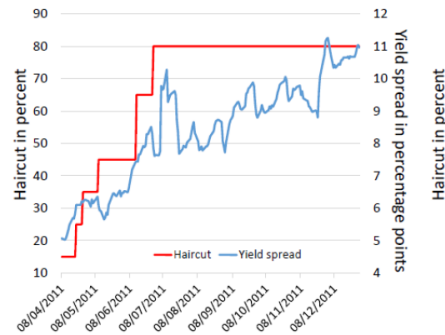


Figure 2: Haircuts and yield spread for 10 year **Portuguese** government bonds on 10 year German Government bond. Source: Armakola, Douady, Laurent and Molteni (2016)

In the case of the 10 year *Portuguese bond*, when margins went up from 35% to 45% on the 11th of May 2011, the yield fell 4% that day (followed by smaller reductions in the next 3 days) and one month later, when LCH.Clearnet Ltd. announced on the 28th of June that margins were being raised to 80%, yields fell by 4% that day, by 2% the next day and by 1% the following date (with further decreases into July) .

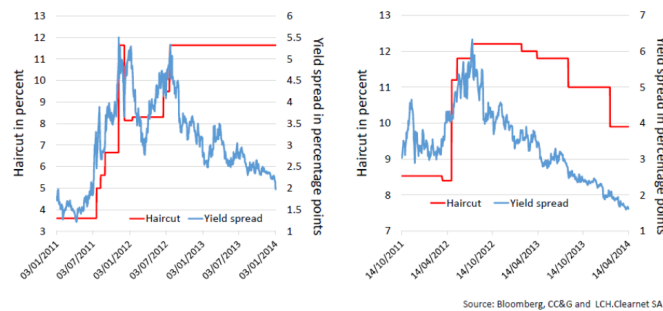


Figure 3: Haircuts and yield spread for 10 year **Italian** (left) and **Spanish** (right) government bonds on 10 year German Government bond. Source: Armakola, Douady, Laurent and Molteni (2016)

For the 10 year *Italian bond*, we observe that margins rose (from 6.65% to 11.65%) on the 9th of November 2011 and yields fell by 4.5% and 6.4% on the 10th and the 11th of November, respectively. After a period of lower haircuts, margins were back at the same high level on the 23rd of July 2012 and yields fell by 1.8%, 6.2% and 1.6% on the 25th, the 26th and 27th. For the 10 year *Spanish bond*, we see that the announcement of a margins update (to 11.8%) made on the 19th of June 2012, with effect on the 21st, led to yield reductions of 2%, 5%, 2% and 3% in the announcement date and the next three dates, respectively.

Other factors may have impacted on yields. Margin increases took place at times when agents' (and credit rating agencies') perceptions of default risk may have fluctuated, affecting also the yields. Sometimes prices go down after margin being raised, as is noticeable for the Spanish bond from April to June 2012 and for the Portuguese bond throughout April 2011. However, this evolution is certainly not the rule. Even for Ireland and Portugal, where, in the first half of 2011, at a first glance, both yields and haircuts seem to have moved around upward trends, when we take a closer look, we see that there are so many downward bumps in yields happening just after rises in margins, that one suspects something almost mechanical at work coming from leverage adjustments.

We could have looked at other examples but the particular example is not our focus, the function of the example is mostly illustrative. Our point is theoretic and simplified for emphasis. It is not our purpose to try to adhere to those illustrative empirical examples closely. However, we see the above mentioned counter-cyclical episodes as *suggesting* that the speculative leveraged positions on these four sovereign bonds can be a factor explaining why prices may rise in response to higher margins. With this empirical motivation in mind we proceed to a theoretical analysis of the mechanisms that could possibly be at work and explain such a behaviour.

4 The Model

4.1 An agent's problem

We consider a binomial economy with two dates. At an initial date (date 0) there is only one node in the event tree, followed by nodes U and D at the second date.

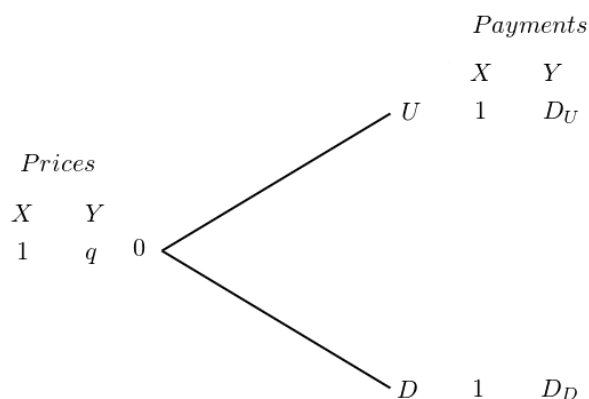


Figure 4: Events tree of the binomial economy.

This economy has just one good. Agents are endowed with, trade and value it only at the second date, in states U and D . We normalize its price to 1. At the first date, agents have initial holdings and trade two assets, X and Y . Asset X pays one unit of the good in each state and asset Y pays D_s units of the good in state $s \in \{U, D\}$, with $D_U < D_D$. The market for these assets opens only at date 0. The price of asset X is normalized to 1 and the price of security Y is denoted by q . There are two possible interpretations. The good can be seen as a perishable consumption good (and X as a productive resource) or we can identify the good with asset X (thought of as a durable good or money).

There is a continuum of agents indexed by $i \in (0, 1)$. All agents have the same endowments: an unitary endowment of each asset at the initial node and state contingent endowments ω_s of the good at the second date. Agents only differ in

the weights that their linear utilities attribute to consumption C_U^i and C_D^i in the U and D states, respectively:

$$(1) \quad U^i(C_U^i, C_D^i) = \gamma^i C_U^i + (1 - \gamma^i) C_D^i,$$

We suppose that $\gamma : (0, 1) \rightarrow (0, 1)$ is a continuous and increasing function.

At date 0, agents pledge the security Y in a *repo* trade. This transaction consists in buying the security and promising to resell it at next date at a predetermined price. There is a difference between the price at which the title is bought, in the first leg of the transaction, and the price at which it is resold to its original owner, in the second leg of the transaction, at the second date. This difference is captured by the *repo rate*.

The repo trade can be seen as a secured loan: in the first leg, one agent pledges the security to get a loan that should be repaid in the second leg, at an interest rate (the repo rate) that was locked in before. Agents take positions x^i and y^i on asset X and security Y , respectively, and at the same time can pledge ψ^i units of the security in repo (in exchange for obtaining a loan with the same value $q\psi^i$) or accept θ^i units of the security as collateral (in exchange for giving a loan with the same value $q\theta^i$). We assumed repo contracts are settled at the security maturity date¹². In this simple case, in the second leg, the loan is repaid (at the predetermined repurchase price) but what is actually given back to the borrower is just the asset payoffs.

A non-negativity constraint applies to X , repo long and repo short positions:

$$(2) \quad x^i, \theta^i, \psi^i \geq 0$$

¹² a.k.a repo to maturity ... alternatively, a 3 dates variant of the model could be worked out easily. JM confusing to reader

Security positions are bounded by the interaction with repo positions. What can be pledged as collateral cannot exceed the security long position. So far, this is would be a traditional collateral constraint. But we go beyond, as the *collateral can be reused* by the creditor: can be pledged in another repo trade or can be sold. Selling the collateral that is temporarily in the creditor's hands is what a *short sale* is. Naked short sales, unrelated to the amount of the security that was borrowed (accepted as collateral), are not allowed in reality and are ruled out in our model. Hence, security position can be negative y^i but short sales are bounded by the net repo position. The *box constraint* captures the two-sided interaction between security and repo positions¹³:

$$(3) \quad y^i + \theta^i - \psi^i \geq 0$$

In other words, the net security title balance held by each agent (in each agent's "box") must be non negative. If the reuse of the collateral were not allowed (as in mortgages), this constraint would collapse to a plain collateral constraint of the form $y^i - \psi^i \geq 0$.

Repo trades are centrally cleared by an exchange (also known as central clearing counterparty, CCP), that charges a margin to both sides of the market, in a proportion $1 - h$ of the value of the repo trade¹⁴. Then, agent i budget constraint at date 0 is as follows:

$$(4) \quad (x^i - 1) + q(y^i - 1) + q(\theta^i - \psi^i) + (1 - h)q(\theta^i + \psi^i) \leq 0$$

¹³ See Bottazzi, Luque and Páscua (2012) on this constraint and also Duffie (1996) for precursors of this constraint.

¹⁴ See Bottazzi, Luque and Páscua (2012) on the OTC case and how haircuts are different in that case.

Or equivalently,

$$(5) \quad (x^i - 1) + q(y^i - 1) + q(2 - h)\theta^i - qh\psi^i \leq 0$$

At the second date, agents settle their repo obligations, using their endowments and the returns from their first date security (long or short) positions. Repo loans are repaid at the repo rate ρ . The CCP gives back to the agents the margins it had collected, accrued of interest, at the same rate, the repo rate

The budget constraint for agent i in state s is as follows:

$$(6) \quad C_s^i = \omega_s + x^i + y^i D_s + (1 + \rho)q(\theta^i - \psi^i) + (1 + \rho)q(1 - h)(\theta^i + \psi^i) \geq 0$$

That is,

$$(7) \quad C_s^i = \omega_s + x^i + y^i D_s + (1 + \rho)q[(2 - h)\theta^i - h\psi^i] \geq 0$$

A plan $(x^i, y^i, \theta^i, \psi^i)$ is said to be *feasible* for agent i at prices (q, ρ) if it satisfies the budget constraints (5) and (7), the box constraint (3) and the non-negativity restrictions (2). A plan that maximizes (1) among all plans that are feasible for i at (q, ρ) is said to be *optimal* for i given (q, ρ) .

4.2 Equilibrium

The CCP collects margins from both sides of the repo market and is supposed to pay back these margins, at the repo interest rate. To accomplish this, it should invest the margins and there is no better way than to invest also in repo, that is, to be also a repo long (accept the security as collateral and provide a loan, on its own), with a position $\Theta^{EX} \geq 0$.

The market clearing condition for the repo market is:

$$(8) \quad \int_0^1 \theta^i di + \Theta^{EX} = \int_0^1 \psi^i di,$$

What the CCP invests in repo is the margins it collects from both sides of the market, that is,

$$\Theta^{EX} = (1 - h) \cdot \left(\int_0^1 \theta^i di + \int_0^1 \psi^i di \right)$$

Hence, repo markets clear if

$$(9) \quad (2 - h) \int_0^1 \theta^i di = h \int_0^1 \psi^i di$$

The market clearing conditions for the two asset markets are:

$$(10) \quad \int_0^1 x^i di = 1 \text{ and } \int_0^1 y^i di = 1$$

An **equilibrium** consists in prices (q, ρ) and an allocation of agents' plans $(x^i, y^i, \theta^i, \psi^i)_{i \in (0,1)}$ such that **(i)** each agent's plan is optimal for him at these prices and **(ii)** asset and repo markets clear¹⁵.

5 Short markets versus long markets

5.1 Leverage

Leverage in securities market is usually measured by the ratio of the value of position to the down payment needed to build that position. In the case of a long position, the purchase of the security can be financed in the repo market -

¹⁵ Second date market clearing follows from asset market clearing and the fulfillment of second date budget constraints as equalities (due to monotonicity in preferences).

pledging the *whole* long position and obtaining a cash loan with the same value, but incurring the cost of paying the margin to the CCP, that is, spending just a fraction $1 - h$ of the security value. Leverage is given by $1/(1 - h)$. Analogously, in the case of a short position, if the agent is short selling the *whole* collateral that he has accepted, it presumes that he gave a cash loan of the same value and paid a margin to the CCP. Again, the net cost is just the fraction $1 - h$ of the security value being shorted. So, leverage is once more given by $1/(1 - h)$. However, in both cases, this is what leverage is when the agent has decided to *lever up to full potential given the haircut*. Lower leverage would result if the long position were not entirely pledged or if the short position were lower than the whole collateral that was accepted.

Collateral reuse may play also a role in allowing for the longs to lever up to full haircut potential. The aggregate (fully) leveraged long position, which is inversely proportional to the haircut $1 - h$, may exceed what the aggregate initial holdings were for this security. Such outcome can only satisfy market clearing if short sales are done in equilibrium. However, how much leverage can be achieved depends both on how wealthy agents are and how low the haircut is.

In equilibrium and for the market at large we will say that the market is long if net trade of longs is more sensitive to leverage than short. The market is short if higher sensitivity is with shorts. In the examples that we present in this paper we exhibit equilibria where the longs or the shorts are levered up to full potential but not both at the same time, hence providing clear examples of ‘long’ and ‘short’ markets. The side hitting that full leverage constraint - only one side does- will tell us whether the *market is long or short*, respectively. In both cases, being fully leveraged means that the security position (long or short, respectively) can be built to the most that the margin downpayment allows for (by combining the

first date budget and box binding equalities)¹⁶. Second date endowments should not be an obstacle to such leverage. That is, in the case of a short market, state D endowments should be high enough (or margins not too low) to allow for the shorts to fully lever while still having non-negative income (not becoming insolvent) at that state, where security returns outweigh repo repayments. Similarly, in the case of a long market, state U endowments should be high enough to allow for the longs to fully lever without becoming insolvent in the state where security returns fall below the repo repayment that is due.

Finally, we note that in both examples, the collateral gets reused. In the ‘short’ case this is obvious but we observe also that in the ‘long’ case short sales are always done in equilibrium. The longs’ wealth (their initial holdings of the two assets, evaluated at equilibrium prices) is high enough (by comparison with what the haircut is), so that aggregate long positions exceed the given aggregate initial holdings.

We will show next that, in our economy, whether the market is long or short is what determines how increases in leverage (reductions in margin) affect the price of the security.

We also identify conditions that give rise to short and long markets. Short markets are likely to occur when the endowment in the state that short sellers value less (ω_D in this economy) is high relative to the endowment in the other state. When ω_U is high enough relative to ω_D , a long market is more likely to occur.

¹⁶ Moreover, there can not be an equilibrium where both the longs and the shorts are fully leveraged, as shown in Appendix A.2.

5.2 Leverage and prices

More formally, let us focus on equilibria where all shorts take the same short position and all longs take the same long position, as will be the case in the examples presented below. We refer to such equilibria as *typical equilibria*. There will be a *marginal agent* m who is indifferent between buying or selling security Y and we denote a typical agent in the set $(m, 1)$ by H and a typical agent in the set $(0, m)$ by L . The former is a pessimist and will be a short. The latter is an optimist and will be a long. **One can show from FOC wrt y that when m increases q decreases**

Definition 1. *We say that a typical equilibrium $(q, \rho, x, y, \theta, \psi)$ is a short (long) market equilibrium if the net trade of a short is more (less, respectively) elastic with respect to the loan-to-value ratio h than the net trade of a long.*

In other words, in a short market it is the shorts who are more sensitive to haircuts, to changes in leverage conditions. They they behave as being the ones that are more leveraged. The elasticity of the net trade of a short with respect to h measures what is the proportional change in the absolute value of the equilibrium net trade $(y^H - 1)$ of a short in response to some proportional change in h , by allowing all equilibrium variables to adjust to this change (that is, *mutatis mutandis*). It is given by $\frac{d \ln(1-y^H)}{d \ln h}$, as $y^H < 0$. This should not be confused with the partial elasticity $\frac{\partial \ln(1-y^H)}{\partial \ln h}$, which assumes the security price and the repo rate to remain constant (a *ceteris paribus* elasticity). Similarly, the elasticity of the net trade of a long with respect to h measures how sensitive is the absolute value of the net trade of a long to changes in h and is given by $\frac{d \ln(y^L-1)}{d \ln h}$, for $y^L > 1$.

Our first result establishes that the way how security prices respond to margins, in typical equilibria, depend on whether markets are long or short.

Proposition 1. *The security price q increases with the loan-to-value ratio h in a*

long market but decreases with h in a short market.

Proof. Market clearing requires $my^L + (1 - m)y^H = 1$. As the loan-to-value ratio h changes, m has to change so that 1 remains a convex combination of the new positions y^L and y^H . More formally,

$$(11) \quad m = \frac{1 - y^H}{y^L - y^H} = \frac{1}{1 + \frac{y^L - 1}{1 - y^H}}$$

We see that m is increasing with the ratio $\frac{1 - y^H}{y^L - 1}$ of the absolute value of the net trade of a short $(1 - y^H)$ to the net trade of a long $(y^L - 1)$. Hence, m increases (and, therefore, q falls) with h if and only if the logarithm of this ratio increases with h , that is, if and only if $\frac{d \ln(1 - y^H)}{dh} > \frac{d \ln(y^L - 1)}{dh}$, or equivalently, if and only if $\frac{d \ln(1 - y^H)}{d \ln h} > \frac{d \ln(y^L - 1)}{d \ln h}$, as claimed. And q increases with h if and only if $\frac{d \ln(1 - y^H)}{d \ln h} < \frac{d \ln(y^L - 1)}{d \ln h}$. \square

In the next subsections we examine how short or long markets may arise and see how security prices get affected by margins in both cases.

5.3 Short market

We construct an equilibrium with a short market. This type of equilibrium only occurs when ω_D is large enough so that what ends up constraining the short position of agents $i \in (m, 1)$ is not the non-negativity of C_D^i but instead the binding box constraint. The shorts sell all their endowments of assets X and Y to build up the largest possible short position in Y . We find that the positions of a

pessimist agent H as follows:

$$(12) \quad y^H = -\theta^H = -\frac{1+q}{q} \cdot \frac{1}{1-h}$$

$$(13) \quad \psi^H = x^H = 0$$

$$C_U^H, C_D^H > 0$$

Equation (12) shows that when it is the box constraint that bounds short-sellers' positions, their positions are inversely related to the haircut, that is given by the leverage $1/(1-h)$. We will see that this is a short market as when margin is raised shorts need to cover more position than longs.

Denote a typical agent in the set $(0, m)$ by L . This is a buyer of both X and Y . From market clearing in the security we see that the security long position of this agent must be the following:

$$(14) \quad y^L = \frac{1}{m} \left[1 + (1-m) \frac{1+q}{q} \cdot \frac{1}{1-h} \right]$$

If the agents that are long in Y are pledging it ($\psi^L > 0$) and not doing any reverse repo ($\theta^L = 0$), the repo market clearing condition (9) requires

$$(15) \quad \psi^L = \left(\frac{2-h}{h} \right) \left(\frac{1-m}{m} \right) \frac{1+q}{q(1-h)}$$

The box constraint requires $y^L - \psi^L \geq 0$, which holds if and only if

$$(16) \quad h/2 \geq (1-m) \frac{1+q}{q}$$

There will be a slack in the box of the non-levered agent L if (16) holds as a strict

inequality. The other positions of agent L satisfy

$$(17) \quad C_U^L = 0, C_D^L > 0, x^L = \frac{1}{m}$$

Agents in $(0, m)$ are moving all their consumption from state U to state D , that is, $C_U^L = 0$.

To be sure that these plans are optimal for each agent, we write agent i 's optimization problem in terms of three choice variables, y^i , θ^i and ψ^i . We denote $E^i D = \gamma^i D_U + (1 - \gamma^i) D_D$, which is agent i 's expected payment of security Y . On the right of each restriction we write the corresponding Lagrange multiplier.

max

$$E^i \omega + (1 + q) + (E^i D - q)y^i + q(2 - h)\rho\theta^i - qh\rho\psi^i$$

s.t.

$$x^i = (1 + q) - qy^i - q(2 - h)\theta^i + qh\psi^i \geq 0 \quad (\mu_x^i)$$

$$y^i + \theta^i - \psi^i \geq 0 \quad (\mu^i)$$

$$C_s^i = \omega_s + (1 + q) + (D_s - q)y^i + q(2 - h)\rho\theta^i - qh\rho\psi^i \geq 0 \quad (\lambda_s^i)$$

$$\theta^i \geq 0 \quad (\nu_\theta^i), \quad \psi^i \geq 0 \quad (\nu_\psi^i)$$

The first order conditions (FOC) of the problem of agent $i \in (0, 1)$ are:

$$y^i : \quad E^i D - q = q\mu_x^i - \mu^i - \sum_s \lambda_s^i (D_s - q)$$

$$\theta^i : \quad q(2 - h)\rho = q(2 - h)\mu_x^i - \mu^i - \nu_\theta^i - q(2 - h)\rho \sum_s \lambda_s^i$$

$$\psi^i : \quad qh\rho = qh\mu_x^i - \mu^i + \nu_\psi^i - qh\rho \sum_s \lambda_s^i,$$

Notice that an agent may have both θ^i and ψ^i positive only if the multiplier μ^i for

the box constraint is zero.

Given that X pays a null interest rate, we look for equilibria where the repo rate is zero as well¹⁷. For agents i , with $i > m$, the first order conditions are satisfied for $\rho = 0$ with the following multipliers:

$$\begin{aligned}\mu_x^H &= \frac{q - E^H D}{q(1 - h)} > 0 \\ \mu^H &= q(2 - h)\mu_x^H > 0 \\ \nu_\theta^H &= 0 \\ \nu_\psi^H &= 2q(1 - h)\mu_x^H \\ \lambda_U^H &= \lambda_D^H = 0\end{aligned}$$

When $i < m$, the multipliers that satisfy the first order conditions for $\rho = 0$ are:

$$\begin{aligned}\mu_x^L &= \mu^L = 0 \\ \nu_\theta^L &= \nu_\psi^L = 0 \\ \lambda_U^L &= \frac{E^L D - q}{q - D_U} > 0 \\ \lambda_D^L &= 0\end{aligned}$$

Shorts have positive shadow values for both the box constraint (a possession value of the collateral, due to the desire to short sell) and the non-negativity constraint on X (expressing a wish to do a "naked short sale" in X). If the former were positive and the latter zero, the repo rate would have to be negative, below the zero interest rate on X .

From these conditions, it follows that if agent $m \in (0, 1)$ is indifferent between

¹⁷ In Fostel and Geanakoplos (2012) the secured interest rate was also zero. If X had stochastic returns, the repo rate could fall in between the implied interest rates on X , interpretable as interest on reserves (IOR).

buying or selling Y the following condition must hold:

$$(18) \quad E^m D = q$$

Assumption 1: We assume that $\omega_U = 0$ but ω_D is high enough so that $C_D^H \geq 0$, which holds if $\omega_D \geq (1 + D_D) \left(\frac{D_D - (2-h)D_U}{D_U(1-h)} \right)$.

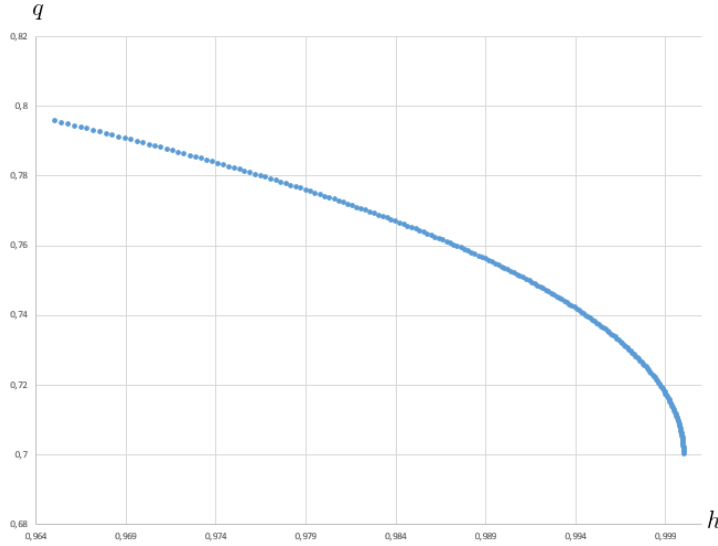
Agents' beliefs are given by $\gamma^i = i$. The equilibrium price q for the security must satisfy $0 = C_U^L = 1 + q + (D_U - q)y^L$. This implies that leverage is

$$(19) \quad \frac{1}{1-h} = \left[\frac{m(1+q)}{q-D_U} - 1 \right] \frac{q}{(1+q)(1-m)}$$

where $q = E^m D = mD_U + (1-m)D_D$. Solving for m as a function of h allows us to find the equilibrium price q of the security, for different values of h . By combining equations (18) and (19) we see that the security price q decreases with h :

Proposition 2. *If commodity endowments satisfy Assumption 1, then the market is short and the security price decreases as leverage goes up (and therefore as margins decrease).*

We present the proof in Appendix A.1 and prefer to present here a numerical illustration. Consider an economy where $D_U = 0.70$, $D_D = 1.15$. As leverage $\frac{1}{1-h}$ increases, the price of the security goes down. Figure 5 shows how *the equilibrium security price decreases with the loan-to-value ratio h* , along the black curve. Condition (16) should hold in equilibrium, this is the case in the region below the gray curve. Equation (19) is depicted by the black curve (when (16) holds) and its dashed continuation (when (16) is not met).

Figure 5: q vs. h - Short market

We have shown that for given security payoffs (D_U and D_D), there are plenty of margin coefficients compatible with equilibria (all haircuts $1 - h$ associated with points on the black solid curve of Figure 5). Higher loan-to-value ratios (h) imply lower security prices, in these short market equilibria.

6 Concluding remarks

We adapted the binomial model in Fostel and Geanakoplos (2012 and 2015) to repo markets, where securities trades can be financed and through which securities may be shorted. The repo party pledges a security to get a cash loan. The reverse repo counterparty proving the loan can reuse the collateral by short selling it. Repo loans are full recourse. For non-recourse loans and non-reusable collateral, leverage always makes the collateral price increase. In our self-collateralised securities market case, such implication holds only if the corresponding security market is ‘long’ - meaning people long the security are more leveraged than those

who are short. On the contrary, if the security market is ‘short’, more leverage pushes the security price down. In Fostel and Geanakoplos (2012 and 2015) the asset market would always be long in our terminology as leverage is only provided to purchase the asset.

We do not think the recourse/non-recourse aspect is key to drive our conclusion (repo market is typically full recourse) On the other hand, rehypothecation to the extent that it is a precondition of even being able to short is a key difference. When leverage can be taken on both the long and short side what matters is on which side leverage is used. The adjustment mechanism goes as follows. Looking at the impact of raised margin requirements separately for agents with a long or a short position in the asset, leverage reduction will mean reduction of position for both. The price of the asset is the balancing factor that restores market clearing. If the position of a short is reduced by more than the position of a long - in what we call a ‘short’ market - the price of the security will go up. The reverse happens when the market is ‘long’ and it has been the case studied by Fostel and Geanakoplos (2012), as this is the only possible outcome for the price of a house securing a mortgage, or for the price of other collateral used to asset back securities.

Positioning (market ‘long’ or ‘short’) is the driver of whether the price of the security will go up or down when leverage is adjusted.

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A Appendix

A.1 Comments on the short market equilibria

1) *Comparing two equilibria:* To gain more intuition, let us see look at equilibria generated by two different haircuts, 1% and 3%. Figures 6 and 7 show the security and repo positions for the continuum of agents in the two equilibria.

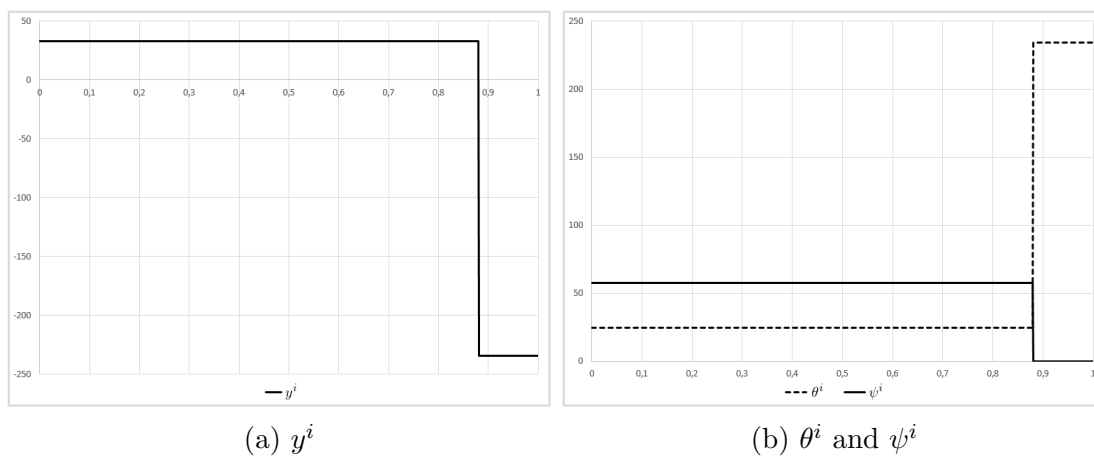


Figure 6: Equilibrium corresponding to haircut of 1% ($h = 0.99$): $m = 0.881$, $q = 0.754$, $x^i = 1/m$ for $i \leq m$ and $x^i = 0$ for $i > m$ (assuming $\omega_D \geq 107.6$).

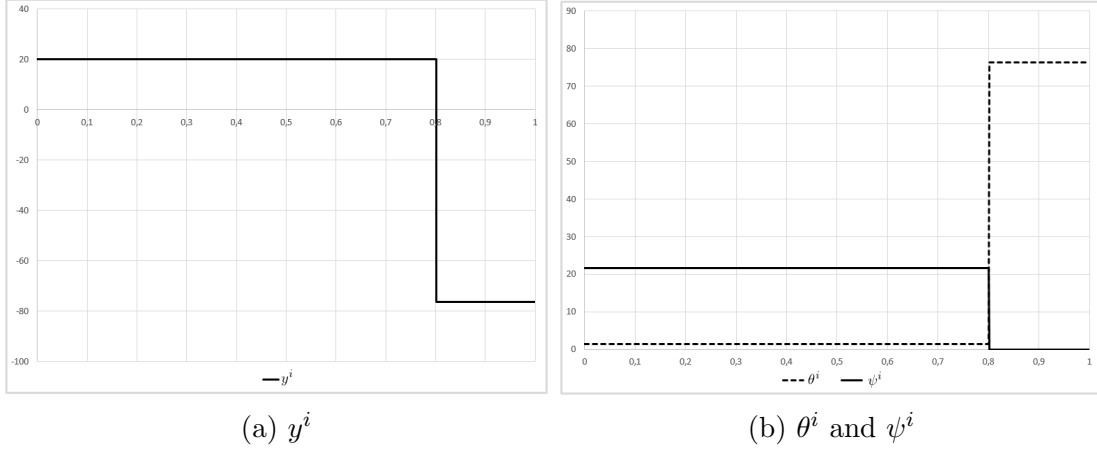


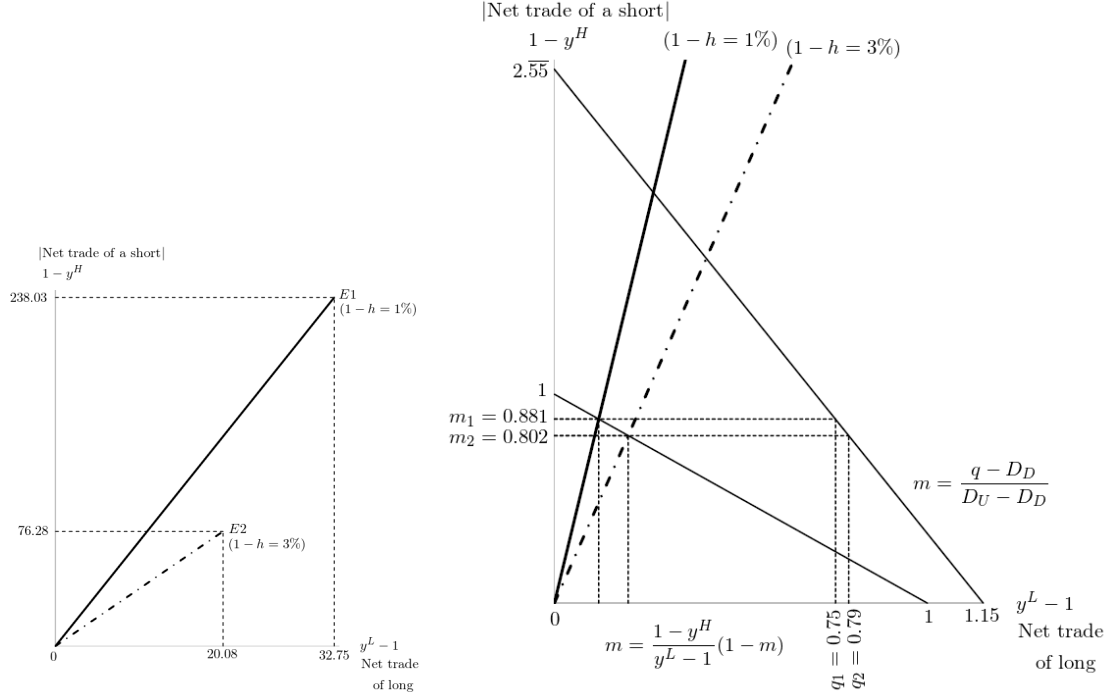
Figure 7: Equilibrium corresponding to haircut of 3% ($h = 0.97$): $m = 0.801$, $q = 0.789$, $x^i = 1/m$ for $i \leq m$ and $x^i = 0$ for $i > m$ (assuming $\omega_D \geq 34.7$).

An agent above the marginal agent ($i > m$) is short selling the whole collateral that was pledged to him. An agent below the marginal agent ($i < m$) is long in the security but pledging less than the long position.

Market clearing requires $my^L + (1-m)y^H = 1$. As the haircut increases, m has to change so that 1 remains a convex combination of the new positions y^L and y^H . Now, the graphs on the left-hand-side of Figures 6 and 7 show that, as the haircut is raised, the fully leveraged (individual) short position $|y^H|$ falls much more than the (individual) long position y^L . The short position y^H changes from -232.7 to -75.6 (is 32% of what it was before), while the long position y^L changes from 32.7 to 20.0 (is still 61% of what it was before). Both y^H and y^L got closer to 1, but the former much more than the latter. The weights on the convex combination must reflect that: m falls and, therefore, $q = E^m D$ rises.

We saw in the proof of Proposition 1 that m is increasing with the ratio $\frac{1-y^H}{y^L-1}$ of the absolute value of the net trade of a short ($1 - y^H$) to the net trade of a long ($y^L - 1$). In a short market, this ratio falls as the haircut is raised. Figure 8 illustrates how m and q vary as this ratio changes. In the first graph we see that this ratio is the slope of a ray joining the origin to the point representing net

trade positions of the two typical agents. The second graph takes an amplified look at what happens closer to the origin, where m can be found as the vertical coordinate of the point where the ray crosses the simplex. Such m induces $q = D_D - (D_D - D_U)m$, read on the horizontal axis.



(a) Net trades of longs and shorts (b) m and q determined from net trades ratio

Figure 8

2) *On other repo positions for the non-levered agents:* as the non-levered longs have null shadow values for the box constraint, the above security trades could be implemented with other repo positions for the longs (and the CCP). Each long could take positions $\tilde{\theta}^L$ and $\tilde{\psi}^L$ on both sides of the repo market and have no slack in the box ($\tilde{\theta}^L = \tilde{\psi}^L - y^L$). This is not the same as pledging the long position in exchange for the net repo loan. Being on both sides of the market entails an extra margin, which will be given back by the CCP in the second leg. That is, the long is at the same time lending cash to the CCP (at the repo rate), thereby undoing

partially his secured borrowing. What marks the difference between levered and non-levered agents is not so much whether there is no slack in the box but rather how tied up is the security position (long or short) to the opposite repo position (short or long, respectively).

Such alternative repo trades would not change how q relates to h . Market clearing requires both $(2 - h)(1 - m)\theta^H + (2 - h)m\tilde{\theta}^L = hm\tilde{\psi}^L$ and $my^L = 1 + (1 - m)|y^H|$, where $|y^H| = \theta^H$. We get $\tilde{\psi}^L = \frac{2-h}{1-h} \cdot \frac{1}{2m}$. It is easy to see that $\tilde{\theta}^L \geq 0$ (and $\tilde{\psi}^L \geq y^L \geq \psi^L$) if and only if (16) holds and, therefore, equilibrium prices q are still related to h according to the solid black curve of Figure 5.

3) *On netting of clients repo positions*: we have assumed that the repo short and long positions of each agent do not get netted for the purposes of assessing marginable positions. However, our results are robust to full or partial netting¹⁸. As shown in Appendix A.3, conditions (16) and (19) still hold under full or partial netting (and, therefore, also Figure 5). The only interesting difference is that under full netting, repo portfolios of non-levered agents will be uniquely determined, for each haircut: non-levered agents can no longer have alternative repo portfolios with positions on both sides of the market (those discussed in Comment 2)).

A.2 Long market

A long market is more likely to occur when ω_U is large relative to ω_D . The equilibrium is again characterized by a marginal agent $m \in (0, 1)$. The set $(m, 1)$ of agents has a typical agent denoted by H and the set $(0, m)$ has a typical agent denoted by L . Now, the agents with high valuations (the optimists) of the expected return of security Y have long positions on the security and are leveraged, leading

¹⁸ LCH.Clearnet does partial netting (see LCH.Clearnet SA (2011)): the marginable position is a repo long or short position decremented by the following: the minimum of the two multiplied by a factor significantly less than one.

naturally to a long market as can be seen below.

$$(20) \quad y^L = \psi^L = \frac{1+q}{q} \cdot \frac{1}{1-h}$$

$$(21) \quad \theta^L = x^L = 0$$

$$C_U^L, C_D^L > 0$$

while agents with low valuations of security returns are short and not leveraged,

$$(22) \quad y^H = \frac{1}{1-m} \left[1 - m \cdot \frac{1+q}{q} \cdot \frac{1}{1-h} \right]$$

$$(23) \quad \theta^H = \frac{h}{2-h} \frac{m}{1-m} \frac{1+q}{q(1-h)}$$

$$(24) \quad \psi^H = 0$$

$$(25) \quad x^H = \frac{1}{1-m}$$

$$C_U^H > 0 \quad C_D^H = 0$$

The box constraint of the non-levered shorts holds if and only if $y^H + \theta^H \geq 0$, or equivalently, for

$$(26) \quad h/2 \leq 1 - m \frac{1+q}{q}$$

The problem of the agents is the same as in the short market example. For the positions presented above, the first order conditions are satisfied when agents' positive multipliers are the following: $\mu_x^L = \frac{E^L D - q}{q(1-h)}$, $\mu^L = qh\mu_x^L$, $\nu_\theta^L = 2q(1-h)\mu_x^L$ and $\lambda_D^H = \frac{q - E^H D}{D_D - q}$.

We will consider the case when the parameters of the economy are all as before with only one exception: we interchange the values of the endowments ω_U and ω_D . Let $D_U = 0.70$, $D_D = 1.15$. We assume now

Assumption 2: $\omega_D = 0$ and $\omega_U \geq (1 + D_D) \left(\frac{D_D - (2-h)D_U}{D_U(1-h)} \right)$ (to ensure that $C_U^L \geq 0$).

Agents' beliefs are again given by $\gamma^i = i$. Now, $C_D^H = 0$ implies that leverage should be such that

$$(27) \quad \frac{1}{1-h} = \left[\frac{(1-m)(1+q)}{D_D - q} + 1 \right] \frac{q}{(1+q)m}$$

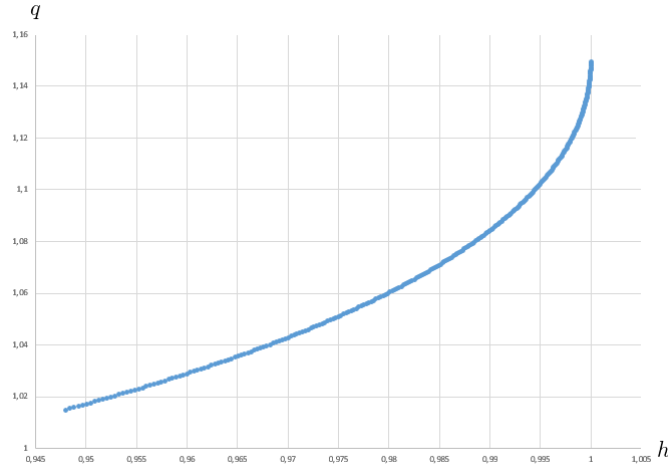
where $m = \frac{D_D - q}{D_D - D_U}$.

In Figure 9, condition (26) holds the region above the gray curve. Combining conditions (27) and (26) we get a lower bound on h compatible with $\psi^H \geq 0$. For h above that lower bound ($h \geq 0.96189$), equation (27) describes how q evolves with h . The black curve in Figure 9 illustrates how the security price increases with the loan-to-value ratio h (and, therefore, with leverage) in the long market¹⁹. This relationship is generalized in the following proposition, proven in Appendix A.1:

Proposition 3. *If commodity endowments satisfy Assumption 2, then the market is long and the security price increases as leverage goes up (and therefore as margins decrease).*

Observe that short sales are always done in equilibrium. In fact, $y^H < 0$ if and only if $m \frac{1+q}{q} \frac{1}{1-h} \geq 1$. By (27), we get $y^H < 0$ if and only if $\frac{(1-m)(1+q)}{D_D - q} + 1 \geq 1$, which holds trivially.

¹⁹ As in the short market examples, we could have allowed now each non-levered short to be just predominantly repo long, rather than exclusively repo long, with no slack in the box, but still with a (short) security position less tied up the (long) repo position, than is the case for the long counterparties.

Figure 9: q vs. h - Long market

A.3 Proof of Propositions 2 and 3

We show that under Assumptions 1 or 2, the security price decreases or increases, respectively, with h , which also implies that the market is short or long, respectively, by the argument in the proof of Proposition 1.

For given a value of h , the equilibrium price of the security must satisfy the following nonlinear system of two equations:

$$(19) \quad H \equiv \frac{1}{1-h} = \left[\frac{m(1+q)}{q-D_U} - 1 \right] \frac{q}{(1+q)(1-m)}$$

$$(18) \quad q = E^m D$$

From equation (19) we see that if we keep q constant and increase m , leverage (H)

increases:

$$\begin{aligned}
\frac{\partial H}{\partial m} &= \left(\frac{1+q}{q-D_U} \right) \frac{q}{(1+q)(1-m)} + \left(\frac{m(1+q)}{q-D_U} - 1 \right) \frac{q}{(1+q)(1-m)^2} \\
&= \frac{q}{(1+q)(1-m)^2(q-D_U)} [(1-m)(1+q) + m(1+q) - q + D_U] \\
&= \frac{q(1+D_U)}{(1+q)(1-m)^2(q-D_U)} > 0
\end{aligned}$$

where the last inequality follows from $q > D_U$, due to (18). Now let $\bar{q} = 1/q$ and note that:

$$(28) \quad \frac{\partial H}{\partial \bar{q}} = \frac{1}{(1-m)(1+\bar{q})^2(1-\bar{q}D_U)^2} [(1-\bar{q}D_U)^2 + m(1+\bar{q})^2D_U] > 0$$

This implies that $\frac{\partial H}{\partial q} < 0$ and that along any isoleverage curve (obtained by fixing a value of H in (19)) we have:

$$\frac{dq}{dm} = -\frac{\frac{\partial H}{\partial m}}{\frac{\partial H}{\partial q}} > 0$$

From $\frac{\partial H}{\partial m} > 0$ we get that, for a fixed value of q , increasing m implies moving to a isoleverage curve corresponding to a higher value of H . The new security price that solves the two-equation system is then found at the intersection of this curve and the straight line given by (18) ($q = D_D + m(D_u - D_D)$). This implies that as leverage H increases, the price of the security q must decrease as shown in Figure 10.

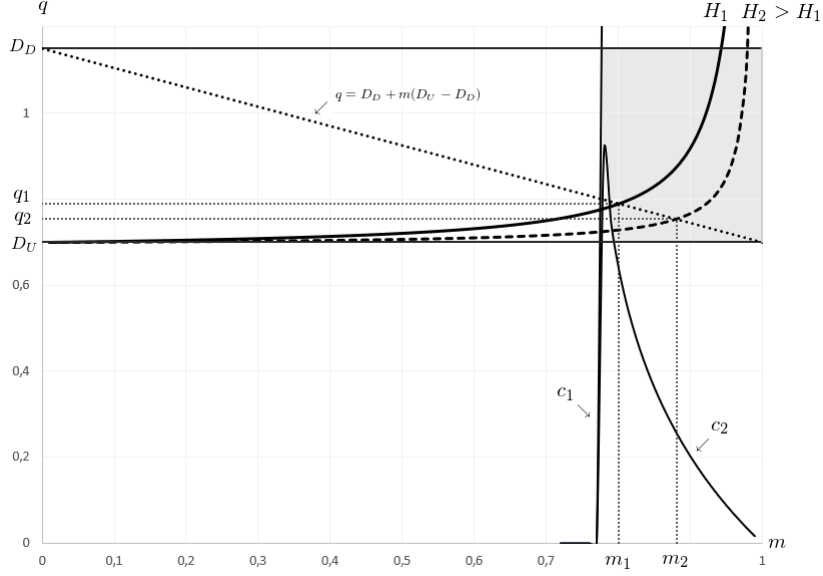


Figure 10: Impact of leverage on equilibrium when the market is short

There is another condition that must hold in equilibrium. It is condition (16), ensuring that the box constraints is satisfied for the non-levered longs. Eliminating h in the system formed by equation (18) and the equality version of (16), we get a quadratic equation in q , for each m , whose two roots are described by curves c_1 and c_2 in Figure 10. The grey region in Figure 10 is the set of points (m, q) where $q \in [D_U, D_D]$ and (16) holds. Equilibrium pairs (m, q) are found within this region where each isoleverage (for some h) crosses the dashed line representing (18). The isoleverages portrayed in the figure are for the haircuts of 1% and 3%, as in the numerical example of section 5.3 (and for its parameter values for D and ω).

An analogous argument proves that, in a long market, prices increase with increases in leverage, as in Proposition 3. To see this simply note that now equilibrium prices must satisfy both (18) and

$$(27) \quad \frac{1}{1-h} = \left[\frac{(1-m)(1+q)}{D_D - q} + 1 \right] \frac{q}{(1+q)m}$$

Defining $\bar{m} = 1 - m$ we have:

$$\frac{\partial H}{\partial \bar{m}} = \frac{q(1 + D_D)}{(1 + q)(1 - \bar{m})^2(D_D - q)} > 0$$

Which in turn implies that now $\frac{\partial H}{\partial m} < 0$. Letting again $\bar{q} = 1/q$ we have:

$$(29) \quad \frac{\partial H}{\partial \bar{q}} = -\frac{1}{m(1 + \bar{q})^2(D_D \bar{q} - 1)^2} [(D_D \bar{q} - 1)^2 + (1 - m)(1 + \bar{q})^2 D_D] < 0$$

This implies that $\frac{\partial H}{\partial q} > 0$ and, therefore, along each isoleverage we have $\frac{dq}{dm} > 0$. Now higher leverage implies moving to a isoleverage to the left. This implies a higher price q for the security and a lower value of m . This is illustrated in Figure 11.

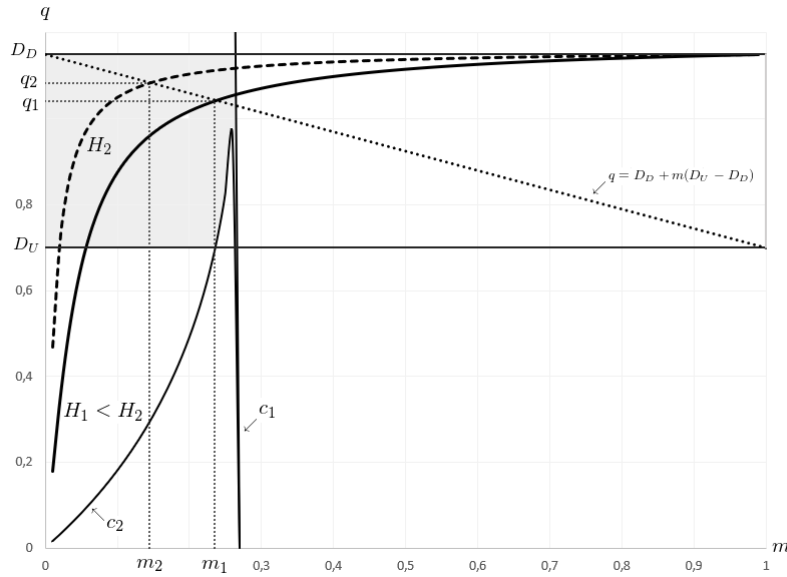


Figure 11: Impact of leverage on equilibrium when the market is long

Equilibrium requires in addition that (26) should hold in a long market. This ensures that the box constraint of the non-levered shorts is satisfied. For $q \in [D_U, D_D]$, such condition holds in the grey region of Figure 11 (constructed from

curves c_1 and c_2 showing where the equality version of (26) holds together with (27)).

A.4 The longs and the shorts can not be both fully leveraged

Suppose we had both short and long agents fully leveraged. This would imply:

$$\theta^H = -y^H = \psi^L = y^L = \frac{1+q}{q} \frac{1}{1-h}$$

Market clearing in the securities market would imply:

$$(1-m)y^H + my^L = -(1-m)\frac{1+q}{q} \frac{1}{1-h} + m\frac{1+q}{q} \frac{1}{1-h} = 1$$

This is:

$$(30) \quad (2m-1)\frac{1+q}{q} \frac{1}{1-h} = 1$$

On the other hand, from equation (9), market clearing in the repo market would imply:

$$(31) \quad \begin{aligned} (2-h)(1-m)\theta^H - hm\psi^L &= 0 \Leftrightarrow (2-h)(1-m) - hm = 0 \\ &\Leftrightarrow m = \frac{2-h}{h} \end{aligned}$$

Combining equations (30) and (31) we get:

$$(2m-1)\frac{1+q}{q} \frac{1}{1-h} = (1-h)\frac{1+q}{q} \frac{1}{1-h} = 1$$

This equation holds only if $1+q = q$, which is impossible for any positive price q .

The reader might be inclined to think that perhaps the fact that the CCP is investing the margin it collects in repo plays a part in this proof but actually it does not make any difference. Suppose that instead of investing in repo, the CCP bought security X . Since both long and short agents are fully leveraged, they would not be buying X . Now market clearing for security X would require:

$$(1-h)[(1-m)\theta^H + m\psi^L] = 1 \Leftrightarrow (1-h)[(1-m) + m]\frac{1+q}{q}\frac{1}{1-h} = 1$$

$$(32) \quad \Leftrightarrow 1+q = q$$

Which is again impossible for any positive price q .

A.5 Netting

Suppose that for given repo and reverse repo positions, ψ and θ , the CCP defines the respective marginable positions, $\hat{\psi}$ and $\hat{\theta}$ as follows

$$(33) \quad \hat{\theta} = \theta - \min\{\theta, \psi\}\alpha$$

$$(34) \quad \hat{\psi} = \psi - \min\{\theta, \psi\}\alpha$$

For $\alpha = 1$ there is full netting, for $\alpha = 0$ there is no netting, while for $\alpha \in (0, 1)$ there is partial netting. To avoid non-convexities, we introduce variables v_1 and v_2 related to marginable positions²⁰

$$(35) \quad (1-h)\hat{\theta} \leq v_1, \quad (1-h)\hat{\psi} \leq v_2$$

$$(36) \quad x = 1 + q[1 - (y + \theta - \psi) - (v_1 + v_2)]$$

$$(37) \quad C_s = \omega_s + 1 + q + (D_s - q)y + \rho q(\theta - \psi + v_1 + v_2)$$

²⁰ We make (35) hold as equalities in equilibrium.

For $\rho = 0$, FOC with respect to θ^i and ψ^i are now given, respectively, by

$$(38) \quad \mu_q^i + (1 - h)\mu_x^i q(1 - \alpha\gamma^i) = \mu^i + \nu_\theta^i$$

$$(39) \quad \mu_q^i - (1 - h)\mu_x^i q(1 - \alpha(1 - \gamma^i)) = \mu^i + \nu_\psi^i$$

where γ^i is 1 when $\min\{\theta, \psi\} < \psi$, is 0 when $\min\{\theta, \psi\} < \theta$ and belongs to $[0, 1]$ otherwise. Take the case of a short market. FOC hold if all multipliers are as in section 5.3 except for ν_ψ^H which is now equal to $q\mu_x^H(1 - h)(2 - \alpha)$. We have $v_1^H = (1 - h)\theta^H$, $v_2^H = 0$, $v_1^L = (1 - h)(1 - \alpha)\theta^L$ and $v_2^L = (1 - h)(\psi^L - \alpha\theta^L)$.

Asset and repo portfolios reported in 5.3 are still equilibrium portfolios and conditions (16) and (19) still hold under full or partial netting (and, therefore, also Figure 5), that is, our result is robust to full or partial netting²¹.

Notice that in the case of full netting, there is no need to use separate variables for repo and reverse repo positions: we can use a signed variable z to denote the net repo trade (reverse repo minus repo) with marginable position defined by $|z|$ (which amounts to having $\alpha = 1$ in formulas (35)). Partial netting seems to be the current practice (LCH.Clearnet stipulates different coefficients α for bonds of different maturities, but always with $\alpha < 1$).

²¹ Moreover, for partial netting, alternative portfolios can again be constructed for the non-levered longs (with positions on both sides of the repo market and no slack in the box) by modifying appropriately those mentioned in Comment 2.