Relational contracts as a foundation for contractual incompleteness
Jean Beuve, Claudine Desrieux

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Abstract
Contractual incompleteness is generally defined by a trade-off between costs and benefits. We examine this trade-off in a dynamic setting and show how the ability of the parties to sustain a relational contract leads to more incomplete contracts.
1. Introduction

Observed contracts are rarely complete in the Arrow Debreu sense. The main reason is that writing complete agreements is costly, and there is a trade-off between these costs and the gains to avoid contractual incompleteness and potential ex post opportunism (Crockers and Reynolds (1993), Battigalli and Maggi (2008)). In this paper, we would like to investigate what happens to this trade-off when parties trade repeatedly. Empirical studies provide various evidence: some contracts become more and more complete over time (Crockers and Reynolds (1993)) and some others are more and more incomplete (Corts and Singh (2004)). We propose here a short model showing that the level of contractual (in)completeness depends on the ability to sustain relational contracts in a dynamic setting. Relational contracts are informal commitments governing non-contractible actions and sustained by the value of future transactions (Bull (1987), Baker et al. (2002)). When the discounted payoff stream from commitment to this informal agreement is higher than the discounted payoff stream from deviation, a relational contract is sustainable and allows to avoid ex post opportunism. Our model shows that in this situation, there is no need to make formal contracts as complete as possible, so that investment in contractual completeness should be lower. This result contributes to the literature on endogenous contractual incompleteness.\footnote{See Kornhauser and Macleod (2010) for a survey about this literature.} It can be related to Tirole (2009) that focus on what drives equilibrium transaction costs when parties have bounded rationality. The role of relational contracts as a factor of contractual incompleteness is suggested in this paper, but as far as we know, our paper is the first model that formally explores such a causality. Only Bernheim and Whinston (1998) have explored the links between incomplete contracts and relational contracts. They regard contractual incompleteness as a cause and not a consequence of relational contracts, since punishment strategies allowing a relational contract to be sustainable can be more easily elaborated when contracts are incomplete. Our contribution is to formally show the reverse causality: incomplete contracts are not a cause but a consequence of relational contracts. The rest of the paper is organized as follows. Section 2 describes our theoretical framework. In section 3, we describe the result under a static framework. Section 4 shows how relational contracting leads to contractual incompleteness in a dynamic framework. Section 5 concludes.

2. The theoretical framework

2.1 Agents and contractual design

We consider a repeated bilateral contractual relationship between a buyer (B, whom we refer as “he”) and a seller (S, whom we refer as “she”). The buyer wishes a project or a service, and asks the seller to perform the work according to his specifications, i.e. according to the contractual design. The value of the project is $K^+$ for the buyer and the seller executes the contract at cost $c$.\footnote{Both $K^+$ and $c$ are common knowledge.} The contract is a
cost-plus contract, so that the seller is paid a price \( P = c + \alpha \) where \( \alpha (>0) \) is the additional compensation beyond the reimbursement of the cost.

As in Bajari and Tadelis (2001), we focus here on problems of \textit{ex post} adaptations in a context where the level of contractual incompleteness is endogenously determined. More precisely, we consider that both parties share uncertainty about contingencies that may arise once the contract is signed and the production begins.\(^3\) Then, during the execution of the contract, some adaptations may be needed to reach \( K^+ \) because the contractual design proved to be inappropriate. In this situation, the contract is said to be incomplete because some actions to reach \( K^+ \) were not foreseen \textit{ex ante}. The parties have then to renegotiate the contract.

2.2 Contingencies

Before proposing the contract, B may perform some costly non-observable efforts to learn about future contingencies, which allows him to propose a more or less appropriate contractual design. As in Tirole (2009), these additional costly efforts incurred before the signature of the contract allow the buyer to determine \textit{ex ante} what may go wrong \textit{ex post} and to draft the contract accordingly. Then, those costs determine the level of (in)completeness.

We denote \( k \in [0; 1] \) the intensity of the effort made by the buyer (at each period) to learn about future contingencies.\(^4\) The higher the intensity of the effort, the more complete the proposed contract will be.\(^5\) Then, by investing \( k \in [0; 1] \):

- With probability \( \rho(k) \), the proposed design (called design A) is the appropriate design. Then, the contract is considered as “complete”, because everything happens as foreseen \textit{ex ante}. The contract delivers utility \( K^+ \) for B and costs the seller \( c \) to produce \( (K^+ > c > 0) \). As a consequence, the utility of the buyer is \( V = K^+ - P \), and that of the seller is \( U = P - c = \alpha \). Hence, the total surplus is \( K^+ - c \).

- But, with probability \( 1 - \rho(k) \), the design is inappropriate and only delivers \( K^- \), with \( K^- = K^+ - \Delta \) where \( \Delta > 0 \). In this case, we consider the contract as incomplete because unforeseen contingencies prevent from reaching \( K^+ \), and parties need to renegotiate their agreement. Indeed, some other, initially unknown, design \( A' \) delivers utility \( K^+ \) to B. Converting A into \( A' \) implies contract’s modifications, that cost “\( a \)” to B. We assume that these costs are distributed over \([a, \bar{a}]\) with \( 0 < a < \bar{a} < \Delta \) according to a probability density function \( z(a) \), and the average value of \( a \) is denoted \( \bar{a} \). The buyer knows this

\(^3\)The seller has no private information about the occurrence of unforeseen contingencies that could arise. See Bajari and Tadelis (2001) to justify this concern for \textit{ex post} adaptation in public procurement. An illustration of such \textit{ex post} adaptation can also be found in Macleod and Chakravarty (2009) about the construction of the Getty museum in Los Angeles.

\(^4\)Since only the buyer may suffer from hold-up in our setting, he is the only party to invest to make the contract more complete.

\(^5\)We speak interchangeably of \( k \) as an effort or an investment in contractual completeness.
distribution.\textsuperscript{6} Then, net gains from renegotiations are $\Delta - a$.\textsuperscript{7} Moreover, the seller can decide to hold-up the buyer during the renegotiation process, \textit{i.e.} she grabs a part $h$ of the net gains of renegotiation. We assume that the seller has an \textit{ex post} bargaining power $\sigma \in [0, 1]$, so that $h = \sigma(\Delta - a)$. As a consequence, the level of hold-up is distributed over $[h, \bar{h}]$ (with $0 < h < \bar{h}$) according to the same probability distribution as $a$.

The function $\rho(k)$ is smooth, increasing, concave, and defined on $[0, 1]$ so that $\rho(0) = 0$, $\rho'(0) = 0$, $\rho'(k) > 0$, $\rho''(k) < 0$, $\lim_{k \to 1} \rho(k) = 1$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{game.png}
\caption{Timing of the game for one contractual period}
\end{figure}

2.3 First-Best level of investments in contractual completeness

Let us determine here the optimal level of investments in contractual completeness $k^*$ that maximizes the total surplus.

\[ k^* = \arg \max_k [\rho(k)(K^+ - c) + (1 - \rho(k))(K^+ - c - \tilde{a}) - k] \iff \rho'(k^*) = \frac{1}{\tilde{a}} \quad (1) \]

The optimal investment is that $\tilde{a}\rho'(k^*) = 1$: the marginal benefit of the investment equals its marginal expected cost.

3. The static game

Let us first suppose that B and S meet only once. Using backward induction, we can easily see that whenever \textit{ex post} adaptations are needed, S decides to hold-up B. Then, the expected payoff of B is $E(V^{NE}) = K^+ - P - (1 - \rho(k))(\tilde{a} + \bar{h}) - k$.\textsuperscript{8}

\[ k^{NE} = \arg \max_k [E(V^{NE})] \iff \rho'(k^{NE}) = \frac{1}{\tilde{a} + \bar{h}} \quad (2) \]

By comparing the first-order conditions (1) and (2), and because of the concavity of the function $\rho(.)$, $\rho'(k^{NE}) < \rho'(k^*) \Rightarrow k^{NE} > k^*$: B over-invests in contractual completeness compared to the optimal level of investment.

\textsuperscript{6}We can assume that the seller also knows this distribution, even if it has no consequence, since she does not bear the cost of these costs of \textit{ex post} adaptation.

\textsuperscript{7}We assume that trade is efficient, \textit{i.e.} $\forall k, a; K^+ - c - (1 - \rho(k))a > 0$.\textsuperscript{8}

\textsuperscript{8}The superscript “NE” stands for “Nash Equilibrium”.

3
Proposition 1. Under a static game, the contract signed between a buyer and a seller is too complete compared to the socially efficient level of completeness.

4. The repeated game

4.1 The dynamic environment

We now consider that the buyer and the seller trade repeatedly. The parties have different discount rates, \( \delta_B \in (0, 1) \) for the buyer, and \( \delta_S \in (0, 1) \) for the seller. These discount rates remain the same for all periods, and are known by the parties.\(^9\) At each end of a period, the buyer can decide to renew the seller or not. We assume that there is no outside option for the seller if the relationship ends, while the buyer can pursue the game with another seller but returns to the Nash Equilibrium level of investment in contractual completeness \( k^{NE} \).

For all \( t \in \mathbb{N}^* \), we denote \( k_t \in [0; 1] \) the intensity of the effort made by the buyer to learn about future contingencies in period \( t \). Since the environment changes over the periods, this effort is specific to each period. Then, at each period \( t \), the design is appropriate with probability \( \rho(k_t) \), and inappropriate with probability \( 1 - \rho(k_t) \).

To sum up, at each period of the game, the buyer has to decide the level of effort \( k_t \), while the seller has decide not to hold-up or to hold-up in case of ex post adaptations, where \( d_t = \{0; 1\} \) denotes this decision. The per-period payoff of the buyer is \( E(V_t) = K^+ - P - (1 - \rho(k_t))(a_t + d_th_t) - k_t \) and that of the seller is \( E(U_t) = P - c + (1 - \rho(k_t))(d_th_t) \).

4.2 The relational contract

We assume that B can propose an informal agreement (i.e. a relational contract) to S and asks her not to hold-up in the case of unforeseen ex post adjustments. This allows him to save on effort \( k_t \). If S cooperates, B promises to renew her with probability 1 at time \( t+1 \). Conversely, if S deviates, B threatens to choose another seller at the next period. If the relational contract is sustainable by both parties, then no hold-up occurs at equilibrium. The level of investment in contractual completeness becomes:

\[
k^{RC} = \arg \max_k [E(V^{RC})] = \max_k [K^+ - P - (1 - \rho(k))\bar{a} - k] \leftrightarrow \rho'(k^{RC}) = \frac{1}{\bar{a}} \quad (3)
\]

In other words, at equilibrium, the level of investment is optimal: \( k^{RC} = k^* \).\(^{10}\) The expected payoff of the seller is \( E(U^{RC}) = P - c = \alpha \) since the seller never holds up.

Let us now see whether such a relational contract can be implemented.

\(^9\) We explore the consequences of asymmetric information on the discount rates in Desrieux and Beuve (2011).

\(^{10}\) This is a stationary equilibrium: \( \forall t \geq 1, k_t = k^{RC} \).
4.3 The participation and self-enforcement constraints of the buyer

The buyer proposes a relational contract only if his expected payoff under relational contracting is higher than under Nash Equilibrium, i.e. if \( E(V_{RC}) > E(V_{NE}) \):

\[
\Leftrightarrow K^+ - P - (1 - \rho(k_{RC}))\tilde{a} - k_{RC} > K^+ - P - (1 - \rho(k_{NE}))(\tilde{a} + \tilde{h}) - k_{NE}
\]

\[
\Leftrightarrow k_{NE} - k_{RC} + (1 - \rho(k_{NE}))\tilde{h} > (\rho(k_{NE}) - \rho(k_{RC}))\tilde{a}
\]

(PCB)

The left-hand side of (PCB) represents the gains of the buyer thanks to the relational contracts: he saves on investments in contractual completeness \( (k_{NE} - k_{RC}) \) and on potential hold-up \( ((1 - \rho(k_{NE}))\tilde{h}) \). The right-hand side of this equation represents the higher cost of contractual modification the buyer is likely to support: because contracts are more incomplete, he will have to finance more frequently the adaptation cost “\( a \)”. Whenever (PCB) holds, the buyer has better propose a relational contract to the seller than choose to over-invest in contractual completeness.

Let us now pinpoint the self-enforcement constraint of the buyer (SEB), i.e. the conditions under which he respects his informal commitment. In case of deviation, he does not renew \( S \) and invests the Nash equilibrium level of investment (with another seller) forever. Then, \( B \) reneges from his informal commitment if:

\[
E(V_{RC}) + E(V_{RC})\delta_B 1 - \delta_B < E(V_{RC}) + E(V_{NE})\delta_B 1 - \delta_B
\]

(SEB)

When (PCB) binds so that \( E(V_{RC}) \geq E(V_{NE}) \), then equation (SEB) never holds: the buyer commits to his informal promise.

**Lemma 1.** When the participation constraint of the buyer holds, a relational contract threatening not to renew the seller in case of hold-up is sustainable by the buyer and allows him to invest \( k^* \), whatever his discount rate \( \delta_B \in (0,1) \).

4.4 The self-enforcement constraint of the seller

The self-enforcement constraint of the seller (SES) implies that her payoff stream is higher under cooperation than deviation (i.e. hold-up and no more trade):

\[
E(U_{RC}) + \frac{\delta_S}{1 - \delta_S}E(U_{RC}) > E(U_{RC}) + h \Leftrightarrow h < \frac{\alpha\delta_S}{1 - \delta_S} \Leftrightarrow \frac{h}{\alpha + h} < \delta_S
\]

(SES)

**Definition 1.** We define \( \bar{\delta} = \frac{\delta_S}{\alpha + h} \) as the discount rate above which the relational contract is sustainable for the seller even for the highest value of hold-up \( (\tilde{h}) \) and \( \tilde{\delta} = \frac{h}{\alpha + \tilde{h}} \) as the discount rate below which the relational contract is never sustainable, i.e. deviation is more profitable even for \( \tilde{h} \).
Following definition 1 and (SES), we can distinguish three seller types: H when \( \delta_S > \delta \), L when \( \delta_S < \delta \), and M when \( \delta_S \in [\delta, \overline{\delta}] \).

**Lemma 2.**

- The type H seller never deviates since her self-enforcement constraint (SES) always holds. The relational contract is sustainable.
- The type L seller always deviates, since deviation is preferable for her even when the smallest amount of hold-up occurs. The SES never holds.
- There is a level of hold-up \( h^M_d \in [h, \overline{h}] \) above which the type M seller prefers to deviate. Following definition 1 and (SES), we can define \( h^M_d = \frac{\delta^M}{1 - \delta^S} \alpha \). The (SES) only holds on \([h; h^M_d]\).

### 4.4 Investment in contractual completeness

From lemma (1) and lemma (2):

- With a type H seller, the relational contract is self-enforced for both the buyer and the seller. The investment in contractual completeness is \( k^{RC} = k^* \).
- With a type L seller, the SES never binds. No relational contract can be implemented, and the buyer has to invest \( k^{NE} \) if he trades with this seller.
- If the seller is of type M, the self-enforcement constraint only binds up to a value \( h^M_d \in [h, \overline{h}] \). As a consequence, the relational contract is not sustainable for all the values of \( h \), i.e. for all value of \( a \). However, under some conditions, the buyer may propose a “second-best relational contract” to the type M seller that allows him to save on the investment in contractual completeness, even if he still over-invests. Let us detail below such a second-best relational contract.\(^{11}\)

A type M seller holds up whenever \( h \geq h^M_d \). Since \( h = \sigma(\Delta - a) \), we denote \( a^M \) the level of the modification cost \( a \) corresponding to \( h^M_d \), so that \( a^M = \Delta - \frac{h^M_d}{\sigma} \). Then, whenever \( a \in [a, a^M] \), the relational contract is no longer sustainable for the type M seller. However, the buyer may still ask the seller not to hold-up and promises him to get an extra bonus when \( a \in [a, a^M] \) if no hold-up occurs. This bonus is an ex ante predetermined payment that depends on the level of \( a \) in case of inappropriate contractual design.\(^{12}\) We denote \( b(a) \geq 0 \) this bonus. Under such a second-best

\(^{11}\)This second-best relational contract is only proposed to type M sellers. Contrary to Desrieux and Beuve (2011), information is symmetric, then there is no strategic behavior from the sellers.

\(^{12}\)Recall that \( a \) is observable by both parties, even if it is non-contractible.
relational contract, the payoffs of the buyer and the type M seller are respectively:

\[
E(V^{SRC}) = K^+ - P - (1 - \rho(k^{SRC}))(\tilde{a} + \int_\tilde{a}^M b(a)z(a)da) - k^{SRC} \\
E(U^{SRC}) = P + (1 - \rho(k^{SRC}))\int_\tilde{a}^M b(a)z(a)da
\]

where \(k^{SRC}\) denotes the level of investment in contractual completeness when the second-best relational contract holds.

Proof 1 in the appendix shows that \(k^{SRC}\) is such that \(\rho'(k^{SRC}) = \frac{1}{\tilde{a} + \int_\tilde{a}^M b(a)z(a)da}\).

By comparing with (1) and (2), we obtain \(k^{RC} = k^* \leq k^{SRC} \leq k^{NE}\).

Proof 2 in the appendix shows that the second-best relational contract is sustainable when:

\[
b(a) = \sigma(\Delta - a) - E(U^{SRC})\frac{\delta_S}{1 - \delta_S} \\
\text{s.t. } b(\tilde{a}) \leq b^{max} = (E(V^{SRC}) - E(V^{NE}))\frac{\delta_B}{1 - \delta_B}
\]

Proposition 2.

- With a type H seller, the buyer’s investment in contractual completeness is at the optimal level \(k^*\) since a relational contract threatening not to renew the seller in case of hold-up is sustainable by both parties.

- With a type L seller, no relational contract is sustainable and the buyer still overinvests in contractual completeness \(k^{NE}\) if he trades with the seller.

- Under some conditions, a second-best relational contract can be implemented between the buyer and a type M seller. It allows the buyer to invest \(k^{SRC}\) so that \(k^* < k^{SRC} \leq k^{NE}\).

5. Conclusion

In this article, we examine how relational contracting determines the level of contractual incompleteness: whenever the buyer anticipates that a relational agreement is sustainable, he knows that he will not be held up, and so invests less in \textit{ex ante} contractual completeness. This allows him to avoid contractual over-completeness observed under a static framework. Our results contribute to the literature on endogenous contractual incompleteness, by stressing another determinant of incompleteness. They also suggest that the identity of the parties matters when they contract, so that an identical transaction can entail different contracting costs (in completeness) depending on the contracting parties involved.
Appendix

Proof 1.

Under a second-best relational contract, the payoff of the buyer when he trades with a type M seller is

\[ E(V^{SRC}) = K^+ - P - (1 - \rho(k))[\bar{a} + (\int_{\bar{a}}^{a^M} b(a)z(a)da)] - k \]

Then, the buyer invests \( k^{SRC} \) in contractual completeness such that:

\[ k^{SRC} = \arg \max_k [E(V^{SRC})] = \max_k K^+ - P - (1 - \rho(k))[\bar{a} + (\int_{\bar{a}}^{a^M} b(a)z(a)da)] - k \]

\[ \iff k^{SRC} \text{ such that } \rho'(k^{SRC}) = 1 + \frac{1}{\bar{a} + (\int_{\bar{a}}^{a^M} b(a)z(a)da)} \]

If the second-best relational contract is sustainable, the payoff of the seller is

\[ E(U^{RC}) = P - c + (1 - \rho(k^{SRC})) \int_{\bar{a}}^{a^M} b(a)z(a)da \]

Proof 2.

Let us now determine the conditions under which this second-best relational contract is sustainable.

The participation constraint of the buyer:

For the buyer to propose a second-best relational contract, his payoff has to be higher under this informal agreement than under Nash equilibrium. His participation constraint (PCB2) is:

\[ E(V^{SRC}) \geq E(V^{NE}) \implies E(V^{SRC}) - E(V^{NE}) \geq 0 \quad \text{(PCB2)} \]

The self-enforcement constraint of the buyer:

The buyer commits to this second-best relational contract when he has better give the bonus \( b(a) \) (when \( a \in [\bar{a}; a^M] \)) than renege and then invests \( k^{NE} \) in the following periods. Then, his self-enforcement constraint (SEB2) is:

\[ \forall a \in [\bar{a}; a^M], \quad K^+ - P - a - b(a) + \frac{\delta_B}{1 - \delta_B} E(V^{SRC}) \geq K^+ - P - a + \frac{\delta_B}{1 - \delta_B} E(V^{NE}) \]

\[ \iff (E(V^{SRC}) - E(V^{NE})) \frac{\delta_B}{1 - \delta_B} \geq b(a) \quad \text{(SEB2)} \]
Let us note that whenever (SEB2) holds, the participation constraint of the buyer (PCB2) binds since:

\[(SEB2) \Rightarrow E(V^{SRC}) - E(V^{NE}) \geq b(a) \frac{1 - \delta_B}{\delta_B} \]

\[\Rightarrow E(V^{SRC}) - E(V^{NE}) \geq 0 \Leftrightarrow (PCB2)\]

To sum up, the buyer can propose a second-best relational contract to a type M seller. This informal agreement foresees to give an extra bonus \(b(a)\) when \(a \in [a, a^M]\) if the seller does not hold up. The buyer proposes and commits to this informal agreement if the bonus \(b(a)\) never exceeds \(b^{max} = (E(V^{SRC}) - E(V^{NE})) \frac{\delta_B}{1 - \delta_B}\).

The highest bonus he has to give occurs when \(a = a\_M\), since it implies \(h = \bar{h}\).\(^{13}\)

In other words, a second best relational contract to be sustainable for the buyer if \(b(a) \leq (E(V^{SRC}) - E(V^{NE})) \frac{\delta_B}{1 - \delta_B}\).

The self-enforcement constraint of the type M seller (SES2):

\[
\forall a \in [a; a^M],
\]

\[
P - c + b(a) + E(U^{SRC}) \delta_S \frac{1}{1 - \delta_S} \geq P - c + h
\]

\[\Leftrightarrow P - c + b(a) + E(U^{SRC}) \delta_S \frac{1}{1 - \delta_S} \geq P - c + \sigma(\Delta - a)
\]

\[\Leftrightarrow b(a) \geq \sigma(\Delta - a) - E(U^{SRC}) \delta_S \frac{1}{1 - \delta_S} \quad (SES2)
\]

A type-M seller commits to the second-best relational contract if he gets a minimal extra bonus \(b(a) = \sigma(\Delta - a) - E(U^{SRC}) \delta_S \frac{1}{1 - \delta_S}\) whenever \(a \in [a; a^M]\).

To sum up, a second-best relational contract that foresees to give to the seller an extra bonus \(b(a)^{14}\) whenever \(a \in [a, a^M]\) can be sustained between a buyer and a type M seller if:

\[
b(a) = \sigma(\Delta - a) - E(U^{SRC}) \delta_B \frac{1}{1 - \delta_B}
\]

\[s.t. \ b(a) \leq b^{max} = (E(V^{SRC}) - E(V^{NE})) \frac{\delta_B}{1 - \delta_B}\]

\(^{13}\)Recall that \(\forall a, h = \sigma(\Delta - a)\)

\(^{14}\)This bonus can be rewritten as \(b(a) + \frac{\delta_S}{1 - \delta_S} (\int_a^{a^M} b(a)z(a)da) = \sigma(\Delta - a) - \frac{\delta_S}{1 - \delta_S} (P - c)\).
References


