Adam Smith on lotteries: an interpretation and formal restatement

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ADAM SMITH ON LOTTERIES: AN INTERPRETATION AND FORMAL RESTATEMENT

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Abstract

The few pages that Adam Smith devoted to lotteries, mainly in the *Wealth of Nations* (1776) did not receive much attention. They nonetheless constituted an opportunity to introduce a sophisticated analysis of individual decision under risk. Through various examples, Smith pointed out a risk-seeking attitude, figured out in the paper in terms of inverse stochastic dominance. However, it is well-known that a contradiction occurs between such an attitude and the principle of an asymmetric sensitivity to favorable and unfavorable events, expressed by a concave function, introduced in the *Theory of Moral Sentiments* (1759). We argue that an appropriate solution to this difficulty should rest on Smith's emphasis on a universal tendency to overestimate the chance of gain, which leads to favor a rank-dependent utility approach within which optimism toward risk can compensate asymmetric sensibility in order to produce some kind of risk-seeking. The question raised by the coexistence of various attitudes toward risk illustrated by the figures of the entrepreneur (typically, the "projector" and the "sober man") gives rise to an extensive analysis, which aims at explaining, on moral grounds, how an initial attitude of risk-seeking can generate prudence before being transformed into risk-aversion.

Keywords: Adam Smith, decision, risk, lotteries, stochastic dominance, rank-dependent utility, asymmetric sensitivity, prudence.

JEL classification: B12, B31, D01, D81.

1. INTRODUCTION

Adam Smith's analysis of behavior under risk might be considered as a by-product of a few pages in a section of a chapter from the *Wealth of Nations* (section 1 of chapter 10, book I), which aims at providing an explanation of income differentials, "arising from the nature of the employments themselves" (*WN*, p. 116)[1]. The whole section of this chapter has

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[1] Adam Smith's works are abbreviated as follows (see complete references in the bibliography): *TMS* = *Theory of Moral Sentiments*; *WN* = *Wealth of Nations*; *LJ* = *Lectures on Jurisprudence* (A and B respectively refer to the manuscript from 1762-1763 and to the manuscript dated 1766).

given rise to reluctant or, on the contrary, to immoderate praise[1]. However, although Smith's emphasis on the part played by risk in the choice of such or such employment was taken quite seriously by his contemporaries (see Bentham's comments in his *Defence of Usury*[2]), it was only considered as leading to a kind of argument which is the "hardest to follow" by modern commentators like A. Rees (1975, p. 343), in order to explain wage differentials. D. Levy (1999), who tried to draw the analytical dimension of Smith's analysis through an expected utility approach, constitutes a notable exception.

An outstanding specificity of these few pages is that they explicitly refer to *lotteries*, which Smith clearly understood in a broad sense, not only as gambles but, also, as various situations of choice. From this point of view, Smith's work might be considered as a landmark in this "classical probability theory" which L. Daston (1988) considered a component of the intellectual project of the Enlightment. As such, classical probability appears at the juncture of two legacies. One of them inherits the seventeenth century mathematical calculus of chance; but the second reaps the reward of a long reflexion coming from medieval lawyers on the degrees of proof, far remote from the calculus of chance in gambles. Jacob Bernoulli's posthumously published *Ars Conjectandi* (1713), especially in part IV on moral, civil, and economic affairs (see I. Hacking 1971), and David Hume's *Treatise on Human Nature* (Hume 1739-40) which discusses philosophical and unphilosophical probabilities in book I, part III, both illustrate the beginning of this tradition in which Adam Smith occupies an eminent place.

The reason is that these pages from the *Wealth of Nations* are an opportunity for Smith to introduce a sophisticated analysis of individual decision under risk (§2). Smith's examples illustrate a relationship between fairness, risk and decision. He uses fairness in order to compare the riskiness of lotteries, pointing out that this usually leads to prefer the riskier ones. Such an attractiveness of risk covers two different ideas, which intuitively correspond to a preference given (i) to a lottery in which the mass of probabilities has moved from the middle to the tails of the distribution (typically, the lottery of the law, when compared to that of the shoemaker) or, more specifically, (ii) to a lottery in which this mass of probability has moved to the right tail from both the middle and the left tail of the distribution (the lottery of the

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[1] On the one hand, see J. Schumpeter: "This is the kind of thing in which A. Smith both delighted and excelled. The lead had been given by Cantillon. But A. Smith went much more deeply into the matter, thus creating an important if not exactly exciting chapter of the nineteenth-century textbook." (Schumpeter 1954, p. 258). And, on the other hand, see M. Blaug: "Until fairly recently, these pages in Smith and a few pages in Marshall's *Principles* exhausted the content of the history of economic analysis of choice among unsure prospects" (Blaug 1962, p. 47). A more qualified point of view can be seen in contributions on risk which favor a historical perspective. P.-C. Pradier, for instance, is clearly aware of the stake of Smith's approach which compares fair and unfair lotteries, but he considers that it is of no consequence at a macroscopic level, and that it is so linked to moral philosophy, that it will not be continued among Smith's followers (see Pradier 2006, pp. 28-30).

[2] See, especially, J. Bentham, letter 13 to Adam Smith, from March 1787, in Smith, *Correspondence*, p. 398.

army, when compared to that of the sea). These two types of lotteries are interpreted hereafter in terms of inverse stochastic dominance.

It is obvious that, in an expected utility framework, this would mean that the underlying valuation function should be convex. But Smith's position concerning asymmetric sensitivity to favorable and unfavorable events, mainly in the *Theory of Moral Sentiments* (*TMS*, p. 144) leads to a concave function (L. Bréban 2012). We argue (§3) that an appropriate solution to this difficulty should rest on Smith's emphasis on a universal tendency to overestimate the chance of gain and, correlatively, to underestimate the chance of loss. This tendency can be interpreted by means of a rank-dependent utility approach, in the line of what J. Quiggin initiated in 1982, where a concave valuation function of the intensity of preferences would be no more inconsistent with a risk-seeking attitude.

At least, such a representation gives consistency to the behaviour of what Smith called the "projector", one of the figures of the capitalist entrepreneur described in the *Wealth of Nations*. But a Smithian theory of decision under risk, only based on the few pages from the *Wealth of Nations* on lotteries, would miss a key feature. According to Smith, the universal tendency to overestimate chances of gain can be challenged by a category of individuals who succeed in transforming their initial risk-seeking attitude into risk-aversion. This last category is typically illustrated by the character of the "sober man", who represents an alternative figure of the capitalist entrepreneur in the *Wealth of Nations*. In order to grasp the attitude of such individuals (§4), we step back to Smith's analysis of prudence in the *Theory of Moral Sentiments*. Following Smith, a prudent behaviour consists in overcoming our natural tendency by adopting the point of view of an "impartial spectator", in order to try to avoid what he calls "hazard" (*TMS*, p. 213) – which denotes any situation involving chances of losses. This last attitude reveals the existence of an attitude toward risk, which we are also used to call "prudence" since the work of M. Kimball (1990). Such prudence is embedded in the properties of the utility function, and can come along with both risk-aversion and risk-seeking. The main lines of Smith's complete theory of behavior under risk can therefore be drawn (§5).

2. Fairness, risk, and lotteries

When commenting on the pages from the *Wealth of Nations* dedicated to lotteries (mainly, I, 10, 1), D. Levy (1999) read Smith through expected utility glasses. Nonetheless, there are good reasons to give up this assumption in favor of a representation more appropriate to Smith's treatment of behavior under risk. And in the early stages, it is even not necessary to have recourse to an alternative representation: within all what Smith says concerning decision under risk, much might be viewed today as a "model-free" discussion – that is, a discussion involving neither expected nor non-expected utility representations.

The basis of this discussion is a curious metaphor through which Smith considers not only gambles strictly speaking, but also decisions concerning the choice of an occupation, insurance or investment, as various types of "lotteries". In the same section 1 of chapter 10, book I, Smith successively introduces: the "lottery of the law" (*WN*, pp. 123-4) which represents a particular case of a lottery of "liberal professions", as compared to the lottery of "mechanic trades" (*WN*, p. 122); public lotteries (*WN*, pp. 124-6); the "lottery of the sea", as compared to the one of the soldier (*WN*, pp. 126-7); and even the "lottery of the church" (*WN*, p. 148). A little further, in the following section on income differentials "occasioned by the policy of Europe" (*WN*, p. 135), he comes again to the question of a lottery of mines (*WN*, p. 187), which he previously dealt with in the *Lectures on Jurisprudence* (*LJ(A)*, pp. 356-7; *LJ(B)*, pp. 494-6), comparing it to a corn lottery, the same question being raised again in chapter 7 on colonies of book IV (*WN*, p. 562).

2.1. Lottery of the state, insurance, and lottery of the law: weak risk-seeking

The lottery metaphor leads Smith to define, at first sight quite accurately, what he calls a "perfectly fair lottery" as a lottery "in which the whole gain compensated the whole loss" (*WN*, p. 125) or, in other words, a lottery where the total outlay is redistributed among gamblers. At the gambler's level, a "perfectly fair lottery" is such that his expected gain equals the price he is asked to pay for gambling. Denoting $L = (x_1, x_2, \dots x_i, \dots\ x_n;\ p_1, p_2, \dots p_i, \dots\ p_n)$ a discrete random variable (a "lottery") where the x_i and p_i are the respective outcomes (ranked in increasing order) and probabilities ($p_1 + p_2 \dots + p_i \dots + \ p_n = 1$) of a state of the world i, and x_0 the outlay,

$$\text{Perfect fairness} \Leftrightarrow x_0 = E(L) \qquad [1]$$

Now, Smith is quite explicit on the fact that the "world neither ever saw, nor ever will see, a perfectly fair lottery" (*WN*, p. 125), and for each example which follows this assertion, he explains which kind of unfair lottery prevails. For instance, he argues that "[i]n the state lotteries the tickets are really not worth the price which is paid by the original subscribers" (*WN*, p. 125). The case of the lottery of the law is quite significant. Smith depicts a "counsellor at law who, perhaps, at near forty years of age, begins to make something by his profession". Had the lottery of the law been a fair lottery, he should have received "the retribution, not only of his own so tedious and expensive education, but of that of more than twenty others who are never likely to make any thing by it". However, Smith explains, "[h]ow extravagant soever the fees of counsellors at law may sometimes appear, their real retribution is never equal to this" (*WN*, p. 123). And many other examples lead to the same conclusion concerning the properties of an unfair lottery – that the price paid to participate in this lottery is greater than its expected outcome:

$$\text{Unfairness} \Leftrightarrow x_0 > E(L) \qquad [2]$$

(Note that Smith does not really take up here the case complementary to [1] or [2], nowadays more familiar, where $x_0 < E(L)$, which could be viewed as another type of unfairness).

When Smith discusses the case of the student in law, of the subscriber to a state lottery, of the neglect of insurance upon shipping or upon houses, of the young man who decides to become a soldier, of the silver mines undertaker, he points out that we are living in a world of unfair lotteries. And the fact that we are inclined towards such unfair lotteries should not be underestimated. It means that facing the alternative “participate in an unfair lottery L and paying for this $x_0 > E(L)$” or “do not participate in this lottery and keeping the outlay x_0 within one’s purse”, many of us would prefer to participate. Since we accept to pay x_0, this means that we prefer L to x_0 and, acknowledging that preferences are monotone, that we prefer L to $E(L)$. In other words (see J. Pratt 1964 and K. Arrow 1965), we are *weakly risk-seeking* (WRS). Denoting $\succ$ the strict preference which constitutes the asymmetric part of the binary relation of preference $\succcurlyeq$ over the set Λ of (possibly degenerated) lotteries and, by abuse of notation, $E(L)$ the lottery which gives a certain outcome equal to the expected value of L, the previous attitude means that

$$\text{WRS: } \forall L \in \Lambda \left(\text{with } L \neq E(L)\right), L \succ E(L) \qquad [3]$$

What might initially be viewed as a judgement about fairness (an unfair lottery is a lottery whose expected value is inferior to the value of the outlay) now appears as an attitude toward risk (WRS) for those who accept, at least potentially, an unfair exchange (pay for a lottery less than the expected value of this lottery). And it is well-known that such an attitude does not require, *per se*, any particular kind of functional representation of preferences over lotteries like, for instance, expected utility or any alternative. That is to say that weak risk-seeking [3] can be seen as *model-free*.

Similar conclusions can be drawn from what Smith says about the demand for insurance. He observes the low proportion of private houses or ships which benefit from insurance:

> Taking the whole kingdom at an average, nineteen houses in twenty, or rather perhaps ninety-nine in a hundred, are not insured from fire. Sea risk is more alarming to the greater part of people, and the proportion of ships insured to those not insured is much greater. Many sail, however, at all seasons, and even in time of war, without any insurance (*WN*, pp. 125-6).

In some occasions, Smith explains, this might be justified because several uncorrelated risks constitute some kind of mutual insurance:

> When a great company, or even a great merchant, has twenty or thirty ships at sea, they may, as it were, insure one another. The premium saved upon them all, may more than compensate such losses as they are likely to meet with in the common course of chances. (*WN*, p. 126).

But usually, such is not the case: most people refuse to pay a “common premium [...] sufficient to compensate the common losses” (*WN*, p. 125), which would make a fair lottery of insurance business[1]. Since the insurance premium is the difference between the value of the good insured and the certainty equivalent of the lottery, it means that this certainty equivalent, x^*, is greater than the expected value $E(L)$ of this lottery. Consequently, the risk premium $\rho(L)$ (the difference between the expected value and the certainty equivalent) should be negative. Now, we know (see, for instance, M. Cohen 1995, p. 75) that an agent for whom

$$\forall L \in \Lambda, \rho(L) < 0 \qquad [4]$$

is also weakly risk-seeking, so that [3] and [4] are equivalent. State lotteries, as well as the demand for insurance, both display a same attitude toward risk.

However, a model-free approach restricted to weak risk-seeking such as expressed in [3] makes the comparison between lotteries a bit flimsy. After all, it only allows comparing a lottery L with a degenerated lottery which gives with certainty an outcome $x < E(L)$, and concluding that L is preferred to x. It allows saying that such *risky* situation is preferred to such other *riskless* situation, but it doesn’t say anything about preferences between different risky situations – that is, between two different non degenerated lotteries L_a and L_b. In other words, *weak* risk-seeking does not say a lot about the intuition of an increase in risk – which M. Rothschild and J. Stiglitz (1970) described as *strong* risk-seeking[2].

2.2. Lottery of the law, again: second-degree inverse stochastic dominance

Smith does not limit his analysis to what we would call “weak risk-seeking”, that is, to the comparison between a risky and a certain outcome. He also proceeds to comparisons between differently risky lotteries. Other examples, still through Smith’s attempt to grasp the meaning of a fair lottery, show that he goes further: he compares unfair lotteries not only to certain outcomes, but also to other unfair lotteries, one of them usually approaching “nearer to a perfectly fair one” (*WN*, p. 125). Again, an initial judgement on fairness leads to a judgement on risk differentials, and at last to the identification of an attitude towards risk. In most of these examples, Smith depicts an individual who prefers unfair to less unfair lotteries: he prefers to try to become a lawyer, rather than a shoemaker (*WN*, p. 124); he buys tickets for a state lottery with “great prizes”, but he neglects these less unfair lotteries where “no prize

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[1] Rigorously speaking, one might argue that there is a difference, at least from the potential insured point of view, since Smith adds that he also has to “pay the expence of management” and a profit at a normal rate (*WN*, p. 125).

[2] The expression is not used as such in the paper, but it corresponds to one of the conceptions of risk which Rothschild and Stiglitz (1970) proved equivalent in an expected utility framework, when comparing lotteries of equal means.

exceeded twenty pounds" (*WN*, p. 125); he chooses to become a soldier, where he could have been a sailor, whereas "the lottery of the sea", Smith says, "is not altogether so disadvantageous as that of the army" (*WN*, 126).

Here again, the case of the "lottery of the law" is a possible key, in order to understand Smith's way of proceeding to compare two lotteries, different according to fairness and to risk. He contrasts the situation of the lottery of the law, and another lottery, of "mechanick trades" or of "common trade, such as that of shoemakers or weavers" (*WN*, p. 123). Let us call these two lotteries respectively $L_a = (x_{a1}, x_{a,2};\ p_{a1}, p_{a2})$ and $L_b = (x_{b1}, x_{b,2};\ p_{b1}, p_{b2})$. For $i = a, b$, x_{i1} and p_{i1} denote the respective *net* outcomes (that is, outcome minus outlay) and probabilities in case of failure, and symmetrically, x_{i2} and p_{i2} are the net outcomes and probabilities in case of success. Of course, $p_{i1} + p_{i2} = 1$. Now, Smith does not only give his reader the required details in order that he understands that the student in law is weakly risk-seeking. He also notices that whereas for a student in law, the probability, to become this highly appreciated counsellor at law is very low, the probability of success is much higher for more common occupations: "Put your son apprentice to a shoemaker", Smith says, "there is little doubt of his learning to make a pair of shoes" (*WN*, p. 122). And in counterpart, he contrasts the "so tedious and expensive education" (*WN*, p. 123) of the student in law, when compared to that of this apprentice shoemaker. His balance between fair and unfair lotteries in the same paragraph (*WN*, pp. 122-3) shows that the lotteries of the law and of the common trade are, respectively, clearly unfair for the first and, if not completely fair, close enough to a fair lottery for the second. In other words,

$$x_{a1} < x_{b1},\ x_{a2} > x_{b2},\ p_{a2} < p_{b2},\ \text{such that}$$

$$E(L_a) < 0 \text{ and } E(L_b) = 0 \qquad [5]$$

But the comparison between the two lotteries is not as simple as it might seem on first view. Through a kind of thought experiment, Smith shifts from the initial unfair lottery L_a to a hypothetically fair lottery of the law (denote it L'_a). Imagining a redistribution in favor of the high outcome, he claims that L'_a is a simple modification of the initial unfair lottery L_a, obtained by an increase in its higher outcome x_{a2}: the expenses of the twenty students in law who failed, Smith says, should be added to the retribution of the one who succeeds (*WN*, p. 123). Since L_a and L'_a are identical except in the case of success, where $x'_{a2} > x_{a2}$, any individual whose preferences are monotone would prefer L'_a to L_a, whatever his or her attitude toward risk. More generally, this means that for each outcome *x*, obtaining at most *x* is more probable with initial lottery L_a than with the modified lottery L'_a, and strictly more probable for at least one *x* (actually, for each $x \in [x_{a2}, x'_{a2}]$).

This can be stated more conveniently in the terms of stochastic dominance[1]. Define a cumulative and a decumulative distribution function as, respectively, $F(x) = \text{Prob}(X \leq x)$ and $\bar{F}(x) = \text{Prob}(X \geq x)$. First degree stochastic dominance (FSD) of a lottery L_F over L_G is defined by $F(x) - G(x) \leq 0$ for all x (with strict inequality for at least one x), and it amounts to saying that any individual whose preferences are monotone prefers L_F to L_G. It is therefore obvious that, if $F_a(x)$ and $F'_a(x)$ are the cumulative distribution functions of L_a and L'_a,

$\forall x \in [x_{a1}, x'_{a2}]$,

$$F'_a(x) - F_a(x) \leq 0 \text{ and } F'_a(x) \neq F_a(x) \Leftrightarrow L'_a \text{ FSD } L_a \qquad [6]$$

And, for this very reason, L'_a is (strictly) preferred to L_a [2]:

$$L'_a \text{ FSD } L_a \Rightarrow L'_a \succ L_a \qquad [7]$$

Turning now to the comparison between L'_a and the lottery of common trade L_b, there is no difference between them concerning their respective fairness, since both of them can be considered as lotteries in which the net outcomes are the results from a redistribution of the total outlays between the gamblers. According to Smith's previous explanations on the cost of the studies in law and on the probability of success, the hypothetical lottery of the law L'_a is a spread (and, since $E(L'_a) = E(L_b)$, a *mean-preserving spread*, MPS) of the lottery of the common trade L_b, since high outcomes are still higher in L'_a, but they are less probable, whereas low incomes are still lower in L'_a, and also more probable:

[1] Stochastic dominance seems to have been introduced in economics after the papers by J. Hadar and W. Russell (1969), G. Hanoch and H. Levy (1969) who analysed first and second-degree stochastic dominance, and M. Rothschild and J. Stiglitz (1970) who presented mean preserving spread as a special case of second-degree stochastic dominance when the expected values of the lotteries are equal. Third-degree stochastic dominance was introduced through a paper by G. Whitmore (1970). Whereas the previous types of stochastic dominance refer to risk aversion, the idea of inverse stochastic dominance, in relation to risk-seeking, appeared in M. Goovaerts *et al.* (1984) and in a paper by K. Zaras (1989) who explored the two types of third-degree inverse stochastic dominance. At the same time, P. Muliere and M. Scarsini (1989) introduced a general analysis of the different degrees of direct and "inverse" stochastic dominance, applied to the question of measures of inequalities. However, "inverse" stochastic dominance is not related here to risk-seeking, since it is not based on decumulative distribution functions $\bar{F}$, but on inverse cumulative distribution functions F^{-1}.

[2] The use of first degree stochastic dominance plays a crucial part in the appraisal of Smith's comparison between an initial (L_a) and a hypothetical modified (L'_a) lottery, because FSD leads to preferences independent from the attitude toward risk. Such is clearly not the case, for instance, when L_a is a state lottery, and L'_a a "lottery in which no prize exceeded twenty pounds", though their probability is higher and L'_a comes closer to a perfectly fair lottery (*WN*, p. 125). This time, the expected value of the lottery is modified not through an increase in the highest outcome, but through a decrease in the outcome and an increase in its probability. It would be easy to check that, in such a situation, L'_a does not first degree stochastically dominate L_a any more, since for all x belonging to $]x'_{a2}, x_{a2}[$, $F'_a(x) - F_a(x) > 0$. However, L'_a second degree stochastically dominates L_a (SSD), because for all x belonging to $[x_{a1}, x'_{a2}]$, the expression $H_2(x) = \int_{x_{a1}}^{x} [F'_a(t) - F_a(t)]\, dt \leq 0$. And Smith rightly concludes from his comparison that "there would not be the same demand for tickets" (*WN*, p. 125). Indeed, although *all* risk-averters would prefer the modified lottery L'_a to the initial state lottery L_a, other people (a majority, according to Smith), among which risk-lovers, would prefer L_a to L'_a.

$$L'_a \text{ MPS } L_b\text{: } x'_{a1} = x_{a1} < x_{b1}\ ; x'_{a2} > x_{b2}\ ; p'_{a2} = p_{a2} < p_{b2}\ ;$$

$$E(L'_a) = E(L_b) = 0 \qquad [8]$$

This intuition is rigorously expressed by the concept of second-degree inverse stochastic dominance (SISD), introduced by M. Goovaerts *et al.* (1984)[1]. The expected values of both lotteries being equal [8], the condition for SISD of L'_a over L_b is given by:

$$\forall x \in [x_{a1}, x'_{a2}],$$

$$\bar{H}_2(x) = \int_x^{x'_{a2}} [\bar{F}'_a(t) - \bar{F}_b(t)]\, dt \geq 0 \text{ and } \bar{F}'_a(x) \neq \bar{F}_b(x) \Leftrightarrow L'_a \text{ SISD } L_b \qquad [9]$$

Though they are identical under the aspect of fairness, the hypothetical lottery of the law is riskier than the lottery of the common trade, in the sense where L'_a is second-degree inverse stochastically dominating L_b. As shown in Figure 1 where the decumulative distribution functions of L_a, L'_a and L_b are represented, condition [9] is satisfied since, because of [8], the area of A + A' is equal to the area of B.

For the individual depicted by Smith, who is getting ready to study law, L'_a, which is riskier than (though as fair as) L_b, would be also (strictly) preferred to it:

$$L'_a \text{ SISD } L_b \Rightarrow L'_a \succ L_b \qquad [10].$$

Since this is a preference for a mean-preserving spread [8], it also expresses attractiveness for the kind of increase in risk described by Rothschild and Stiglitz (1970) as strong risk-seeking (SRS):

$$\text{SRS: } L'_a \text{ MPS } L_b \text{ and } L'_a \succ L_b \qquad [11]$$

This might be interpreted as a change, resulting from Smith's thought experiment, in the starting point of the analysis. Imagine that the initial choice is not between L_a and L_b, but between L'_a and L_b. Allowing continuity of preferences, it is obviously the very existence of a strict preference in favor of the riskier lottery L'_a which leaves room to the possibility of a preference over the lottery of common trade L_b given to some other lotteries, not as good as L'_a:

$$L'_a \text{ SISD } L_b \Rightarrow \exists L_a\text{: } E(L_a) < 0,\ L'_a \text{ FSD } L_a, \text{ and } L_a \succ L_b \qquad [12]$$

A risk-seeking attitude expressed by [10] has now allowed the introduction of an unfair lottery L_a which anybody should consider worse than L'_a, and that the individual described by Smith considers better than L_b.

[1] An equivalent concept, "risk-seeking second-degree stochastic dominance" (RSSD), defined on the basis of cumulative distribution functions, was introduced by H. Levy (2006, p. 126).

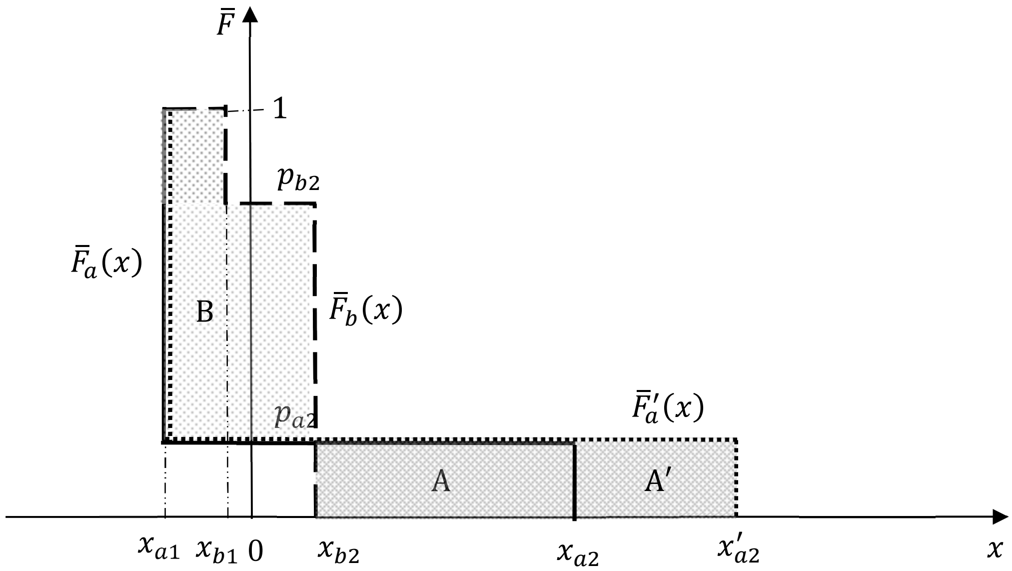

Figure 1 – Decumulative distribution functions: the lottery of the law L_a, the hypothetical fair lottery of the law L'_a, the lottery of common trades L_b. $E(L_a) < E(L'_a) = E(L_b)$; L'_a SISD L_b; L'_a FSD L_a

This contributes toward making explicit the kind of relation that Smith establishes between fairness, risk, attitude toward risk, and preferences between lotteries. The hypothetical fair lottery of the law is just as fair as the lottery of the common trade: both can be viewed as lotteries in which a same total outlay is redistributed among gamblers, although the former is riskier, in the sense of second degree inverse stochastic dominance, than the latter. When this increased risk gives rise to preferences, it also opens the path to the possibility, for *some* lotteries, first degree stochastically dominated by the hypothetical fair lottery of the law, to be nonetheless preferred to the lottery of the common trade. Such is the case for the effective lottery of the law. This latter, now, is clearly less fair than the hypothetical fair lottery of the law and, therefore, also less fair than the lottery of the common trade. In other words, it is not because a lottery is riskier than another that it is also less fair. But it is because a riskier lottery is preferred to a less risky one that there is room for giving preference to an unfair lottery, like this of the law, over a fair or nearly fair lottery, like this of common trades.

The already treated examples of the state lotteries or of the demand for insurance can be interpreted in the same way. If x_0 is the price of the ticket for a hypothetical lottery L' such that $E(L') = x_0$, L' is a fair lottery. Because of weak risk-seeking [3], L' would be preferred to x_0, and it is obvious that L' SISD x_0. Now, this preference given to the riskier option allows the possibility for a lottery L, such that L' FSD L (for instance, diminish the higher outcome in L' to build L), to be preferred to x_0. And because L' FSD L implies $E(L) < E(L') = x_0$, preference is given to an unfair lottery.

2.3. The lottery of the army: third-degree inverse stochastic dominance of the second type

Some, among Smith's other examples illustrate the same approach which makes preference for an unfair lottery an effect of a risk-seeking attitude expressed by second-degree inverse stochastic dominance. Some, but not all, and these complications show that although Smith had risk-seeking in mind, second-degree inverse stochastic dominance is at times insufficiently selective in the characterization of possible risk-seeking attitudes. This leads to introduce some kind of *prudent* risk-seeking attitude, in that it expresses the intuitive idea of an individual all the more (less) risk-seeking that the possible gain (loss) is important. It should be emphasized that this does not mean that this individual would be somehow less risk-seeking, but that he is risk-seeking in a particular way. In standard analysis, for instance, this might refer to an individual who is interested in increasing not only the variance, but also the skewness of a distribution (the second and the third central moment). He would therefore favor a spread to the right tail of the distribution (to increase the variance), and a contraction from the left tail (to increase the skewness). Rigorously, this is represented by third-degree inverse stochastic dominance of the second type (see Goovaerts *et al*. 1984 and Zaras 1989).

An example of such complications appears when Smith moves to the question of knowing why "common people […] enlist as soldiers, or go to sea" (*WN*, p. 126), which gives rise to a comparison between these two unfair lotteries, the "lottery of the sea [being] not altogether so disadvantageous as that of the army" (*WN*, p. 126). On first view, this looks very much like the example of the lottery of the law and the lottery of the common trades – the first one standing for the lottery of the army (denote it again L_a) and the second for the lottery of the sea (L_b). A difference seems to come from the fact that there is a wider range of payoffs for each lottery. Ranking them in increasing order, it means that in each case, one might (i) be killed in action; (ii) stay a simple soldier or sailor; (iii) become an officer; (iv) be promoted at the highest rank.

Smith barely discusses the lowest outcomes. Everyone, among his readers, knows what they are: death in each case, to which Smith refers discreetly when he says that "[w]hat a common soldier may lose is obvious enough" (*WN*, p. 126), adding a little further that the possible positive outcome of the lottery in which he is involved is "the whole price of [his] blood" (*WN*, p. 126); or, about sailors, when he brings up the "dangers and hair-breadth escapes of a life of adventures" (*WN*, p. 127). He is still less explicit concerning this danger, and he does not seem to make any difference concerning the probability of death for a soldier and for a sailor. But for all other outcomes, it seems better to be in the army than in the navy:

> The great admiral is less the object of publick admiration than the great general, and the highest success in the sea service promises a less brilliant fortune and reputation than equal success in the land. The same difference runs through all the inferior degrees of preferment in both. By the rules of precedency a

> captain in the navy ranks with a colonel in the army: but he does not rank with him in the common estimation. (*WN*, p. 126).

Whatever the reasons put forward, they aim at explaining why so many young people prefer to be a general than an admiral, a colonel in the army than a captain in the navy, and a simple soldier than a simple sailor, whose hard life is described in details (*WN*, p. 127). But this difference is challenged by the probabilities of the respective outcomes of the two lotteries:

> As the great prizes in the lottery are less, the smaller ones must be more numerous. Common sailors, therefore, more frequently get some fortune and preferment than common soldiers (*WN*, pp. 126-7).

This means that whereas it is more probable to become an admiral than a general, a captain in the navy than a colonel in the army, it is less probable to remain a simple sailor than a simple soldier.

Now, the difference between the lottery of the army and the lottery of the sea on the one hand, and the lottery of the law and the lottery of common trades on the other hand, becomes more conspicuous. When you move from the lottery of common trades to the lottery of the law, your greater possible gain becomes both higher and less probable. Like when you move from the lottery of the sea to the lottery of the army, where you might become a colonel instead of a captain, a general instead of an admiral. But what for the lower ranks? In the worst case, you die, and it doesn't seem that there is one among the two lotteries in which it would be undoubtedly less probable. If you chose the army, remaining a private is more probable than remaining a sailor in the navy: this still looks like the lotteries of the law and of common trades. But your life would be better as a soldier than as a sailor, and that is contrary to the lowest respective outcomes of the lotteries of the law and of common trades. In the lottery of the law, you exchange the possibility of a greater higher outcome for a smaller probability *and* a smaller and more probable lower outcome. In the lottery of the army, you still exchange the possibility of a greater higher outcome for a smaller probability. But it is not that great, since in counterpart, your lower outcome, though still more probable in the army, is now *higher* and not smaller.

Smith's argument might be captured more easily by simplifying the number of ranks he refers to. Assume that, here again, there are only two possible couples of net outcomes for each lottery L_a and L_b: x_{a1} with probability p_{a1} stands for the outcome of, say, the soldier, x_{a2} with probability p_{a2} for that of the general; x_{b1} with probability p_{b1} stands for the outcome of the sailor, x_{b2} with probability p_{b2} for that of the admiral (for *i* = *a*, *b*, $p_{i1} + p_{i2} = 1$).

To sum up, the respective values of the outcomes and probabilities in the lotteries L_a and L_b are such that:

$$x_{a1} > x_{b1}, x_{a2} > x_{b2};\ p_{a2} < p_{b2},$$

$$E(L_a) < E(L_b) < 0 \qquad [13]$$

The fact that either the lottery of the army or the lottery of the sea might be chosen although they are unfair, and although other occupations, closer to a fair lottery, are available, shows that the individual concerned is weakly risk-seeking.

Nonetheless, carrying on with the same approach as with the lotteries of the law and of common trade does not provide what might have been expected. Let us transform the lottery of the army in the same way, from L_a to L'_a, that is, to a hypothetical lottery of the army where the retribution of the general is increased till it becomes as fair as the lottery of the sea. It is obvious that, here also, [6] and [7] are satisfied so that L'_a FSD L_a, and would be universally preferred to the initial lottery of the army (everybody who aims at becoming a general would prefer to be a better paid general). Turning to the respective characteristics of L'_a and L_b, they are now:

$$x'_{a1} = x_{a1} > x_{b1}\ ;\ x'_{a2} > x_{a2} > x_{b2}\ ;\ p'_{a2} = p_{a2} < p_{b2}\ ;$$

$$E(L'_a) = E(L_b) < 0 \qquad [14]$$

At the difference of the lotteries of the law and of the common trade, the graphs of the decumulative distribution functions $\bar{F}'_a$ and $\bar{F}_b$ in Figure 2 clearly show that L'_a cannot be second-degree inverse stochastically dominating L_b: as a result of [14], A + A' < B so that condition [9] is not fulfilled[1]. However, provided the lotteries do not give comparatively too high outcome to the soldier or too low outcome to the general, and given the equality of the expected values of both lotteries [14], conditions of third-degree inverse stochastic dominance of the second type (TISD2; see Goovaerts *et al.* 1984, Zaras 1989) from L'_a on L_b are satisfied:

$$\forall x \in [x_{b1}, x'_{a2}],$$

$$\bar{H}_3(x) = \int_x^{x'_{a2}} \bar{H}_2(t)\,dt \geq 0 \text{ and } \bar{F}'_a(x) \neq \bar{F}_b(x) \Leftrightarrow L'_a \text{ TISD2 } L_b \qquad [15]$$

The intuitive meaning of TISD2, where $\bar{H}_3(x) \geq 0$ in [15], is that L'_a results from a spread of the high outcomes of L_b and a contraction of its low outcomes, so that risk is both increased and transferred from lower to higher outcomes. And the individual who strictly prefers enlisting in the army to joining the navy is risk-seeking in the sense of TISD2:

$$L'_a \text{ TISD2 } L_b \Rightarrow L'_a \succ L_b \qquad [16]$$

Like in the case of the lottery of the law, the equalization of fairness between the hypothetical lottery of the army and the lottery of the sea allows preference for the riskier. And it is, again, in this interval, between both lottery, that there is room for some lotteries first-degree

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[1] For the same reason, third-degree stochastic dominance (TSD) of L'_a on L_b is excluded.

stochastically dominated by L'_a, like the initial lottery of the army L_a, which can be preferred to that of the sea:

$$L'_a \text{ TISD2 } L_b \Rightarrow \exists L_a\text{: } E(L_a) < E(L_b), L'_a \text{ FSD } L_a, \text{ and } L_a \succ L_b \quad [17]$$

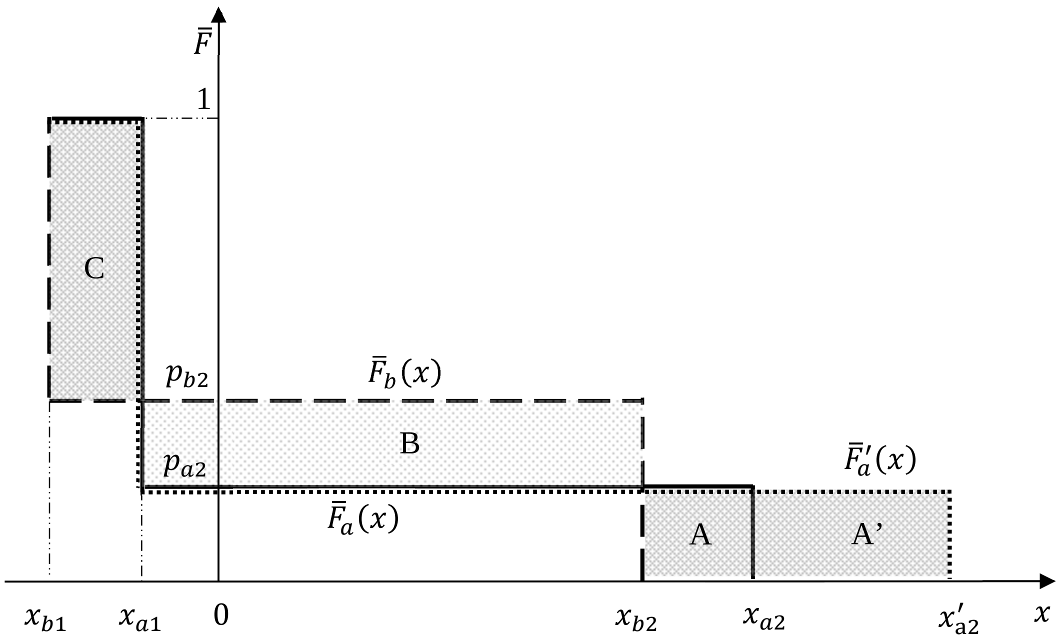

Figure 2 – Decumulative distribution functions: the lottery of the army L_a, the hypothetical fair lottery of the army L'_a, the lottery of the sea L_b. $E(L_a) < E(L'_a) = E(L_b)$; L'_a TISD2 L_b; L'_a FSD L_a

2.4. Limits to a model-free approach

What Smith says about fair and unfair lotteries has opened the path to a model-free interpretation of the attitude toward risk of those who prefer unfair lotteries, in terms of second-degree inverse stochastic dominance, or third-degree inverse stochastic dominance of the second type. Yet, giving up this model-free approach through what seems the easiest way, an expected utility representation, would be rather intuitive: stochastic dominance establishes meaningful relations between the different kinds of dominance and the properties of the underlying function which gives rise to expected utility. Consider the following convex sets of underlying functions:

$U^1 = \{u(x): u' > 0\}$ (increasing functions)
$U^{12} = \{u(x): u' > 0, u'' \geq 0\}$ (increasing convex functions)
$U^{123} = \{u(x): u' > 0, u'' \geq 0, u''' \geq 0\}$ (increasing convex functions with non-decreasing convex first derivative)

L_a and L_b are two lotteries, whose expected utility are noted, respectively, $EU(L_a)$ and $EU(L_b)$. The following results allow linking an attitude toward risk, expressed by stochastic dominance, and preferences over lotteries, expressed by expected utility[1]:

(1) If L_a FSD L_b, then $EU(L_a) \geq EU(L_b), \forall u(x) \in U^1$ [18]a
(Hadar and Russel 1969; Hanoch and Levy 1969)

(2) If L_a SISD L_b, then $EU(L_a) \geq EU(L_b), \forall u(x) \in U^{12} \subset U^1$ [18]b
(Goovaerts *et al*. 1984)

(3) If L_a TISD2 L_b, then $EU(L_a) \geq EU(L_b), \forall u(x) \in U^{123} \subset U^{12} \subset U^1$ [18]c
(Goovaerts *et al*. 1984; Zaras 1989).

The conclusion seems straightforward. Since SISD and TISD2 are required in order to give an account of the attitudes toward risk which Smith points out when he moves to the analysis of lotteries, the expected utility hypothesis leads to represent choices by means of the corresponding functions belonging to U^{12} or to its subset, U^{123}.Typically, this would mean that, in the case of the choice between the lottery of the law and that of the common trade, where SISD prevails, all the increasing convex functions, and no increasing concave function, would be possible candidates[2]. They would all express risk-seeking, through a notion which makes equivalent the various concepts of risk attitude which might be distinguished in a model-free context (see Rothschild and Stiglitz 1970): convexity of the utility function is equivalent to strong risk-seeking, to weak risk-seeking, and to the existence of a negative risk premium. In the case of the choice between the lottery of the army and that of the sea, TISD2 just leads to reduce to a subset U^{123} of U^{12} the set in which any utility function might represent it. Obviously, the already noticed equivalences would still hold, but with the supplementary characteristic that risk-seeking is increasing with *x*. Therefore, it seems that the only remaining elementary problem is to make sure that among such a wide range of possibilities, one of them at least is not contradicted by what Smith says about decision.

The point is that the range of possibilities is still too narrow, so that none of them fits Smith's position.

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[1] A systematic account of results concerning the links between stochastic dominance, direct and indirect till degree 3, and the properties of the utility function can be found in Zaras (1989) and (except for TISD1 and TISD2) in H. Levy (2006), chap. 3 who provides both necessary and sufficient conditions. General results concerning direct stochastic dominance of degree *n* are established by P. Fishburn (1976).

[2] Imagine a concave function allows $EU(L'_a) > EU(L_b)$. This would imply strong risk aversion (SRA) and therefore, that a lottery which is a mean preserving spread (MPS) of another lottery cannot be preferred (see Rothschild and Stiglitz 1970). Now, L'_a is a MPS of L_b, so that it would contradict the preference given to L'_a over L_b. As a result, no concave function is a possible candidate to represent preferences generated by SISD through an expected utility approach.

3. The "absurd presumption": over- or under-valuation of chances

In the *Theory of Moral Sentiments*, Smith argued in favor of an asymmetric sensitivity to favorable and unfavorable events (*TMS*, p. 44). It is tempting to interpret this in terms of a "loss aversion principle", *à la* D. Kahneman and A. Tversky (1979), like in N. Ashraf, C. F. Camerer and G. Loewenstein (2005). Nonetheless, as Bréban (2012) pointed out, this interpretation (and, more generally, a reference-dependent approach to Smith) lacks textual evidence. In this case, asymmetric sensitivity rather refers to a function which plays the same part as a utility function whose propriety of concavity or convexity would depend on a position within a social scale of happiness: when an individual's permanent state of happiness is close to the highest level, the function is concave, otherwise, it is convex. And since in most cases, according to Smith, people are closer to the highest level of happiness, the function is usually concave: a favorable event is assumed to have a smaller impact on enjoyment than the symmetrical unfavorable one. This means that $u(x)$ should be an increasing *concave* function:

$$u(x) \in U_2^1 = \{u(x): u' > 0, u'' \leq 0\} \quad [19]$$

(Bréban 2012)

As a result, $u(x)$ can obviously belong neither to the set of increasing convex functions U^{12} nor, *a fortiori*, to its subset U^{123}. And since risk-seeking, in an expected utility approach, would require the convexity of the underlying function (like in [18]b or [18]c), such an approach seems of no help for giving an account of what Smith says about choice among lotteries. This issue is the one which M. Allais raised as soon as 1953, when criticizing expected utility theory: a same property (concavity or convexity) cannot express both a valuation of preferences (cardinal utility) and an attitude toward risk.

3.1. Can we keep up the expected utility interpretation?

An ingenious way to by-pass this difficulty would be to argue that some other elements in Smith's analysis, left aside till now, lead to question the very relevance of risk-seeking for such situations as the choice of a professional occupation, so that a standard, risk-adverse expected utility approach might be favored.

When comparing liberal professions with mechanic trades, Smith acknowledges various possible reasons for the success of the former. Seemingly, he accepts two of them:

> "First, the desire of the reputation which attends upon superior excellence in any of them; and, secondly, the natural confidence which every man has more or less, not only in his own abilities, but in his own good fortune." (*WN*, p. 123).

Each of these reasons deals with a different matter:

- The “desire of reputation” concerns the motives for action, and comes to explain the success of liberal professions, whereas they “are, in point of pecuniary gain, evidently under-recompenced” relatively to “mechanick trades” (*WN*, p. 123).
- The “natural confidence” deals with individual’s ability to evaluate situations. It also comes to explain the success of liberal professions, although “[i]n the greater part of mechanick trades, success is almost certain; but very uncertain in the liberal professions” (*WN*, p. 122). A closer examination shows that, according to Smith, natural confidence applies to two different topics (*WN*, p. 124):
    - “the over-weening conceit which the greater part of men have of their own abilities”,
    - and “the absurd presumption in their own good fortune”,

  the last one, only, being relevant for his analysis of choices under risk.

Concerning the first issue, the “desire of reputation” allows Smith to explain the success of liberal professions by pointing out that “pecuniary gains” are not the sole reward of such professions, since reputation, or what he also calls “public admiration”, might constitute the greatest part of their outcome:

> The publick admiration which attends upon such distinguished abilities, makes always a part of their reward; a greater or smaller in proportion as it is higher or lower in degree. It makes a considerable part of that reward in the profession of physick; a still greater perhaps in that of law; in poetry and philosophy it makes almost the whole. (*WN*, p. 123).

This position is a long lasting one for Smith, already present, as noted by the editors of the *Wealth of Nations*, in the *Lectures on Jurisprudence*. This might suggest that the lottery of the law is not that unfair and that, possibly, the novice poet or lawyer is not more risk-seeking than the novice shoemaker. Such an interpretation seems to have been favored by D. Levy (1999) who uses the “desire of reputation” argument in order to conciliate an expected utility approach with Smith’s discussion of lotteries.

Our interpretation would be rather that this plays the same part as Smith’s shift to a hypothetically fair lottery, when he imagines that the expenses of twenty unsuccessful students in law are added to the retribution of the only one who succeeds: no need to imagine a fair lottery, it actually exists! This is unambiguously confirmed by what he already said in the *Lectures*:

> The ten or twelve therefore who come into business must have wage(s) not only to compensate the expence of their education, which is very great as a man must be 30 years or thereabouts before he can be of any service as a lawyer, but also the risque of not being ever able to make any thing by it. The temptation to engage in this or any other of the liberall arts is rather the respect, credit, and emin(en)ce it gives one than the profit of it. Even in England where they are more highly rewarded than any where else, if we should compute them according to the same rule as that of a smith or other artizan they would be still rather too low. But the honour and credit which attends on them is to be considered as a part of the wages and a share of the reward” (*LJ*(A), pp. 354-5).

Consider again the lottery of the law: on the one hand, it is now as fair as the lottery of the shoemaker; but on the other hand, it is also a mean preserving spread of this lottery, so that it dominates it stochastically in the sense of SISD. Consequently, the utility function which might be used to represent the choices of the lawyer or of the poet should still belong to the set of increasing convex function: the compatibility between Smith's position and an expected utility approach is not that easy to obtain.

3.2. The "absurd presumption" in one's own good fortune

The first issue – desire of reputation – reinforces the negative argument against the expected utility interpretation: taking it into account confirms that an expected utility framework does not fit to Smith's claims concerning both valuation of preferences and attitude toward risk. Now, the decisive argument is a positive one, related to the second issue – natural confidence – which opens onto an alternative to an expected utility approach. After having distinguished natural confidence as a mental state from the nature of the motives involved in the choices between lotteries, Smith takes care to separate its two possible objects, which both contribute to explaining why a riskier lottery is chosen: natural confidence, either in one's "own abilities", or in one's "own good fortune" (*WN*, p. 124). Moreover, although this last type of confidence might be related to this kind of belief which Hume called "unphilosophical probability" (Hume 1739-40, I, pp. 143-54; see M.-A. Diaye and A. Lapidus 2012), Smith stresses the novelty of his views on the estimation of the probabilities of gains and losses:

> The over-weening conceit which the greater part of men have of their own abilities, is an antient evil remarked by the philosophers and moralists of all ages. Their absurd presumption in their own good fortune, has been less taken notice of. It is, however, if possible, still more universal. There is no man living who, when in tolerable health and spirits, has not some share of it. The chance of gain is by every man more or less over-valued, and the chance of loss is by most men under-valued, and by scarce any man, who is in tolerable health and spirits, valued more than it is worth. (*WN*, pp. 124-5)

In the introductive discussion of the lottery of the law, Smith's claim for novelty appears as resulting from a specific rhetoric. Within this rhetoric, the desire of reputation and the overestimation of one's ability successively vanish in favor of what feeds the analysis of various types of lotteries which runs through this part of chapter 10: the over- or under-valuation of chances. Like the desire of reputation, the first component of natural confidence, the overestimation of one's own ability, does play an important part, since it concerns the consequences of actions (see Bréban 2011, pp. 249-63). However, this is clearly not the issue on which Smith tries to draw his reader's attention. The very mechanism which is at the core of our tendency to prefer unfair and riskier lotteries rests on a propensity to depart from probabilities, that Smith calls the "absurd presumption in [one's] own good fortune".

In spite of its alleged novelty, this mechanism seems quite intuitive: we have a tendency to over-value probabilities of gains, and to under-value probabilities of losses (*WN*, p. 125). Let us then take Smith's claim in a systematic way, and consider again a lottery like this of the army, but without aggregating the different ranks into two categories. We are now facing n different ranks, which give rise to outcomes x_i which increase from x_1 (say, a private), to x_n (a general) with probabilities p_i going similarly from p_1 to p_n. Let us focus on an intermediate rank *i* – a captain, for instance. Because of his "presumption" in his "own good fortune", the candidate willing to join the army over-values his "chance of gain" (*WN*, p. 125): the decision weight $\pi(x \geq x_i)$ which he associates to the possibility of obtaining at least the rank of a captain, is greater than its probability $p(x \geq x_i)$. And consistently to what Smith says about the tendency to under-valuate lower outcomes, the decision weight associated to obtaining a rank lower than this of a captain, $\pi(x < x_i) = 1 - \pi(x \geq x_i)$, would be smaller than its probability $p(x < x_i) = 1 - p(x \geq x_i)$. In case this property holds for all ranks *i* in the army (and assuming that the decision weight of a probability equal to 0 is also equal to 0 and that the decision weight of certainty is, like its probability, equal to 1), it follows closely Smith's argument on over- and under-valuation. Define now an increasing function φ from [0, 1] into itself, which transforms the probability of obtaining an outcome at least as great as x_i into its decision weight. The previous discussion means that:

$$\pi(x \geq x_i) = \varphi\left(\sum_{j=i}^{n} p_j\right) \geq p(x \geq x_i) = \sum_{j=i}^{n} p_j \qquad [20]$$

(equality holds when $p(x \geq x_i)$ equals 0 or 1)

Elementary decision weights π_i can be easily derived from [20] as the difference between the decision weight of obtaining at least x_i, and the decision weight of obtaining at least x_{i+1}:

$$\pi_i = \pi(x \geq x_i) - \pi(x \geq x_{i+1}) = \varphi\left(\sum_{j=i}^{n} p_j\right) - \varphi\left(\sum_{j=i+1}^{n} p_j\right) \qquad [21]$$

$$\pi_n = \varphi(p_n)$$

Always according to [20], φ is such that for all p in [0, 1], $\varphi(p) \geq p$, the equality holding only when p equals 0 or 1. It is obvious that this amounts to saying that φ is strictly concave, that is $\varphi'' < 0$. An immediate consequence of this property of φ should be noted. Imagine two ranks h and k, denoting for instance a sergeant and a colonel, such that $x_h < x_k$ and $p_h = p_k = \bar{p}$. The over-valuation of the chance of becoming a colonel, in comparison of this of becoming a sergeant, implies that $\pi_h < \pi_k$ or, according to [21], that a positive magnitude like its right member, $\varphi(p + \bar{p}) - \varphi(p)$ is decreasing with p (that is, increasing with the outcome, when moving from the sergeant to the colonel): because of the concavity of φ, though it is as probable to become a sergeant as it is to become a colonel, the decision weight of the former is smaller than that of the latter.

This interpretation of Smith's argument matches an idea introduced in economic literature by J. Quiggin (1982), which is at the origin of the variety of models belonging to what had later been known as a "rank-dependent utility" approach[1]. Dealing with cumulated, instead of elementary, probabilities has the advantage of avoiding the major drawback of an intuitive but rather rude representation based on the transformation of elementary probabilities (that is, $\pi_i = \varphi(p_i)$): the well-known possibility that the preferred lottery is first-degree stochastically dominated by a non-preferred lottery[2]. Accordingly, Smith's intuition of a transformation of probabilities leads to a (rank dependant) valuation $U(L)$ of a lottery[3], now ensuring that preference for first degree stochastically dominant lotteries will not be violated:

$U(L) = \sum_{i=1}^{n} \pi_i u(x_i)$ [22]a

where $\pi_i = \varphi\left(\sum_{j=i}^{n} p_j\right) - \varphi\left(\sum_{j=i+1}^{n} p_j\right)$ [22]b

(and $\varphi' > 0; \varphi(0) = 0; \varphi(1) = 1; \pi_n = \varphi(p_n)$)

It is obvious that here, φ plays a decisive part, since it determines the differences between decision weights and probabilities. For instance, strict concavity (which implies $\varphi(p) > p$ when p is different from 0 or 1) or convexity ($\varphi(p) < p$) of φ respectively come into an over-valuation or an under-valuation of the probabilities of high outcomes and, symmetrically, into an under-valuation or an over-valuation of the probabilities of low outcomes, currently interpreted (see Cohen 1995) as, respectively again, "optimism" or "pessimism" under risk (the limit case where φ is both concave and convex, and transforms each probability in itself, corresponding to the situation where the individual is neither optimistic nor pessimistic under risk, and behaves according to a standard expected utility approach). Smith's idea of a specific transformation of probabilities, where those which are related to high outcomes

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[1] Rank-dependent utility now constitutes an important trend in decision theory. For an introduction focusing on associated risk perceptions see, among others, E. Diecidue and P. Wakker (2001), M. Abdellaoui (2009), and M. Cohen (2012). With some qualifications, more recent versions of prospect theory also belong to this kind of models, at least since Tversky and Kahneman's 1992 paper (see Wakker 2010).

[2] Violation of first-degree stochastic dominance (FSD) means that between two lotteries, an individual might prefer the one which gives at least any outcome $\bar{x}$ with a probability lower than the one of obtaining at least the same outcome $\bar{x}$ with the other lottery. It is obvious that a minimum requirement for a theory of choice under risk is the exclusion of the possibility that a first-degree stochastically dominated lottery be preferred, whatever the concerned individual's attitude toward risk, provided his preferences are monotone. This minimum requirement is also lacking not exactly in Kahneman and Tversky (1979), since they use an editing procedure which prevents it, but to the simpler version which Wakker (2010, pp. 274-5) called "separable prospect theory". This issue seems to have had a significant part in the elaboration of Quiggins' ideas on the way probabilities should be transformed. Wakker (2010, p. 153) mentions an unpublished letter by Quiggin on the violation of first-degree stochastic dominance in a paper by J. Handa from 1977 for similar reasons. The argument is resumed in Quiggin (1981, pp. 160-1) and constitutes a basis for his influential article of 1982.

[3] In standard rank-dependent utility models, $U(L)$ is usually written:

$U(L) = u(x_1) + \cdots + \varphi\left(\sum_{i=j+1}^{n} p_i\right)\left[u\left(x_{j+1}\right) - u\left(x_j\right)\right] + \cdots + \varphi(p_n)[u(x_n) - u(x_{n-1})]$,

which is equivalent to [22]a and b (when $\varphi' > 0$; $\varphi(0) = 0$; $\varphi(1) = 1$). In order to keep notations consistent with those of the previous section, we have kept up the use of ranking outcomes in an increasing order, from the lowest outcome x_1 to the highest x_n. Nonetheless, in recent literature, rank-dependent utility models usually favor notations in which outcomes are ranked in a decreasing order.

increase whereas the ones which concern low outcomes decrease, might therefore be understood as "optimism" in this technical sense. It is expressed by:

$$\varphi'' < 0 \qquad [23]$$

Apart from formal aspects, the meaning of φ is far from self-evident, and two alternative interpretations seem to arise. The first one is that the individual concerned has a false perception of probabilities – π_i instead of p_i. The second interpretation is that although his perception of probabilities is correct (he is not mistaken about p_i), he nevertheless considers the different issues with more or less optimism or pessimism (which leads him to set the decision weight π_i above or beneath p_i, in line with the value of $\varphi\left(\sum_{j=i}^{n} p_j\right) - \varphi\left(\sum_{j=i+1}^{n} p_j\right)$). Now, when Adam Smith argues that we over-value or under-value probabilities, he clearly favors the second interpretation. When he discusses the choice of a profession by "young people", he contrasts, on the one hand probabilities ("misfortune" or "good luck"), and, on the other hand, optimism or pessimism ("hope" or "fear"):

> "The contempt of risk and the presumptuous hope of success, are in no period of life more active than at the age at which young people chuse their professions. How little the *fear* of *misfortune* is then capable of balancing the *hope* of *good luck*, appears still more evidently in the readiness of the common people to enlist as soldiers, or to go to sea" (*WN*, p. 126; our italics, L.B and A.L.).

The argument is rather sophisticated. What Smith explains is that focusing only on probabilities (misfortune, or good luck) would lead a young man to move away from the army or from the navy, like from studying law. When he takes fear and hope into account, he does not become ignorant of these probabilities. But this brings out a widely spread discrepancy between decision weights and probabilities, so that the army, the navy, and studying law, can become attractive.

According to this last interpretation, each decision weight conveys the combination of two elements: probability strictly speaking, and the correlative pessimism or optimism under risk (fear and hope, in Smith's words). The resulting attitude toward risk hence combines both the effect of the intensity of preferences (asymmetric sensitivity), expressed through the concavity or convexity of the underlying function $u(x)$, and the optimism or pessimism expressed, again, by the concavity or convexity of $\varphi(p)$. Now, a difficulty arises from the specification of the risk-seeking attitude toward risk which would allow the consistency between the two propositions derived from Smith's writings: the concavity of both $u(x)$ and $\varphi(p)$.

A. Chateauneuf and M. Cohen (1994, p. 89) gave the condition on φ which allows weak risk-seeking (WRS) to prevail, in spite of the concavity of $u(x)$[1]. Since only WRS is concerned, this would support Smith's position on our willingness to buy tickets for an unfair lottery (see above, [3]), but it does not say anything on more complex choices between lotteries, like these of the law and of the shoemaker. Yet, we have dealt with these more complex choices, arguing that they give rise to preference for inverse stochastically dominating lotteries ([12], [17]). But if we keep on considering that for the student in law, for instance, *all* the mean-preserving spread lotteries (like the hypothetically fair lottery of the law) are preferred to the initial lottery (this of the shoemaker), that is, if preference between lotteries equally fair (with equal means) is *always* granted to the second degree inverse stochastically dominating one, which corresponds to *strong risk-seeking* (SRS, [11]), the conclusion would be a bit disappointing: S.-H. Chew, E. Karni and Z. Safra (1987) have shown that in a rank-dependent utility model, SRS is equivalent to the *convexity* of the utility function and the concavity of the function of transformation of probabilities. Like in an expected utility framework, this means that we cannot get rid of the convexity of the utility function, and that SRS is not compatible with Smith' position on asymmetric sensitivity.

The solution rests on a refinement of the involved concept of risk-seeking. It is clear that WRS, which allows consistency between the concavity of u and φ is too demanding since it orders too few lotteries and that SRS (that is, SISD with mean preserving spread), which does not allow consistency, is insufficiently demanding. So that there is room for a subset of the set of pairs of lotteries ordered by SRS which would not forbid the concavity of u. A similar issue would also apply to lotteries ordered by TISD2. In the case of SRS, A. Chateauneuf, M. Cohen and I. Meilijson (2005), identified such a subset, linked to *monotone risk-seeking* (MRS)[2], and gave a condition on u and φ which ensures that in spite of the concavity of u, optimism is high enough to generate (monotone) risk-seeking[3].

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[1] Chateauneuf and Cohen (1994) have established the following condition on u and φ under which an individual is weakly risk-seeking. They first show that if $u(.)$ is increasing ($u' > 0$), assuming the convention that $0 \leq y < x \leq 1$, there exists $h \geq 1$: $u'(y) \leq h((u(x) - u(y))/(x - y))$. Then, if φ is such that $\varphi(p) \geq 1 - (1 - p^h)$ for each p in [0, 1], the individual concerned is weakly risk-seeking.

[2] Monotone risk-seeking results from preference given to a monotone mean preserving spread. A lottery L_a is a monotone mean preserving spread of L_b if there exists a lottery L_c comonotone to L_b with $E(L_c) = 0$, such that $L_a = L_b + L_c$. The idea of an increase in risk is therefore more intuitive (see examples in Cohen 2012) when the mean preserving spread is monotone.

[3] Define first the index of non-convexity of u as $T_u = \sup_{x<y}\left(u'_x/u'_y\right)$ and the index of optimism of φ as $O_\varphi = \inf_{0<P<1}\left(\frac{\varphi(p)}{p}/\frac{1-\varphi(p)}{1-p}\right)$. The condition for the presence of monotone risk-seeking when the utility function is concave might be stated as follows (Chateauneuf, Cohen and Meilijson (2005)). If u is concave (that is, $T_u > 1$), there is MRS if and only if $O_\varphi > T_u$ (if optimism toward risk compensates the concavity of u).

3.3. An illustration

The comparison between the lottery of the law and that of the shoemaker is a typical instance of the possibility to obtain risk-seeking in spite of a greater sensitivity to unfavorable events. It might be viewed as a simple case (only two possible outcomes for each occupation), in which the lottery of the law L'_a is a (monotone) mean preserving spread of the lottery of the shoemaker L_b – provided the desire of reputation (*supra*, p. 17) or, equivalently, the hypothetical fair lottery (*supra*, p. 7) is taken into account – so that they share the same expected outcome ($E(L'_a) = E(L_b)$). This is a typical instance because in the framework of expected utility, preference given to the lottery of the law over the lottery of the shoemaker would require the convexity of the utility function. On the contrary, Figure 3 and Figure 4 below show how, in a rank-dependent utility framework, such a preference can be made consistent with a concave underlying function expressing asymmetric sensitivity to favorable and unfavorable events, like in Bréban (2012).

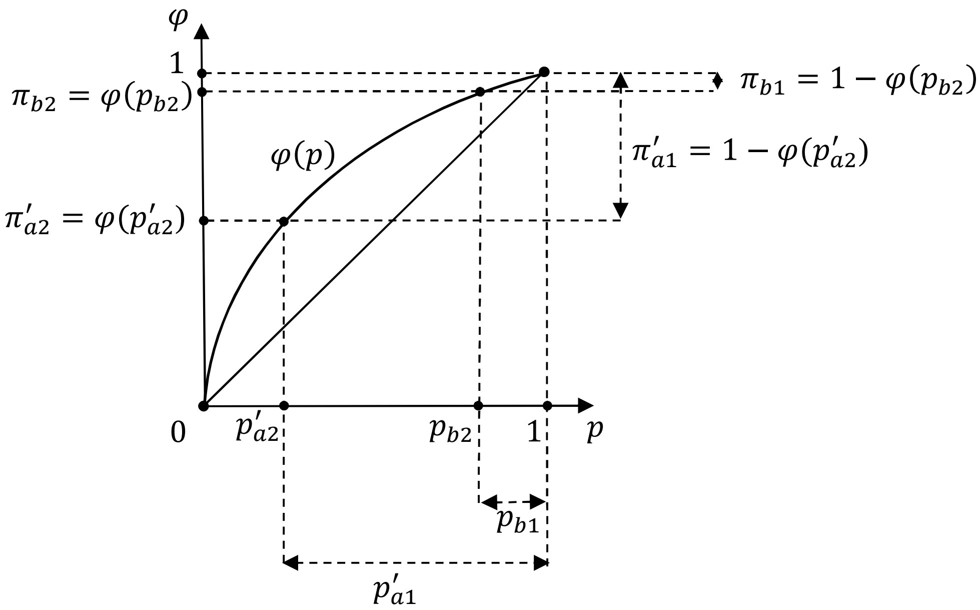

Figure 3 – Lottery of the law and lottery of the shoemaker – The transformation of probabilities by fear and hope

The respective probabilities of success in both lotteries, p'_{a2} and p_{b2}, are transformed (Figure 3) in decision weights π'_{a2} and π_{b2}, the former transformation being more important than the latter, because of the concavity of $\varphi(p)$, so that the decision weight of a success in studying law has proportionally much more increased than this of the success in becoming a shoemaker (and, conversely, for the respective decision weights of failure $\pi'_{a1} = 1 - \pi'_{a2}$ and $\pi_{b1} = 1 - \pi_{b2}$).

In Figure 4, because of the concavity of $u(x)$, the expected utility $EU(L'_a)$ of the lottery of the law is smaller than that of the shoemaker $EU(L_b)$, which contradicts the preference given to the former. But moving to the decision weights given in Figure 3 increases the valuation of L'_a

to the detriment of that of L_b in terms of rank-dependent utility (*RDU*): $RDU(L'_a) > RDU(L_b)$.

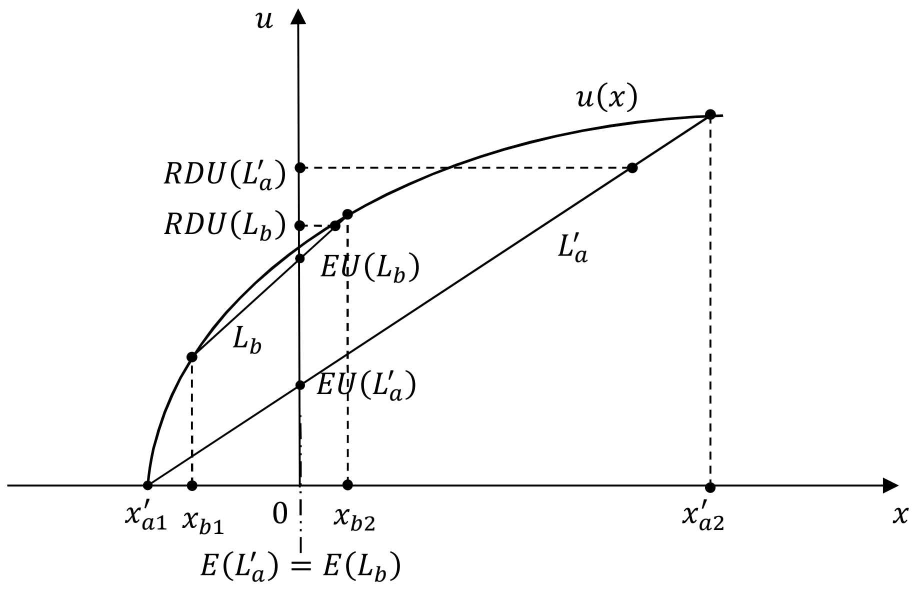

Figure 4 – Lottery of the law and lottery of the shoemaker
– Rank-dependent utility *vs* expected utility valuation

In spite of a greater sensitivity to unfavorable events, preference given to the riskier lottery, like the lottery of the law, or the lottery of the army now makes sense, even if it is second degree (or, in the case of the army, third degree second type) inverse stochastically dominating the other lottery – that of the shoemaker, or of the navy.

4. TOWARD A COMPLETE THEORY OF BEHAVIOR UNDER RISK

This shows the consistency between two kinds of arguments on which Smith's conception of decision under risk relies:

- the first argument, from section 3, part I of the *Theory of Moral Sentiments*, concerns the widely spread asymmetric sensitivity to favorable and unfavorable events, according to which the latter have a more important effect than the former;
- the second argument, from chapter 10, book I of the *Wealth of Nations*, concerns the as widely spread tendency to give preference to a riskier lottery, in the sense of second or some higher degree inverse stochastic dominance.

The solution can be found in a rank-dependent utility approach, which emphasizes a tendency which Smith also presents as universal: the overvaluation of the chances of gain, which comes along with an undervaluation of the chances of loss. As a result, individuals are led to commit in riskier and unfair situations, such as liberal professions, public lotteries or army, whereas less risky and fairer opportunities are available. A great part of former difficulties of interpretation of the few seemingly simple pages on lotteries comes from the fact that an intuitive expected utility approach definitely did not fit them.

4.1. Smith on entrepreneurship: from projectors to sober men

Once this interpretation is acknowledged, it can be extended in order to understand some of the various characters which take place in the *Wealth of Nations*. And among them, this category of capitalist entrepreneurs which Smith calls "projectors" (*WN*, p. 307; p. 357). These latter are involved in operations quite similar to unfair lotteries, needing an important capital and faced with a high risk of bankruptcy, which the amount of profits does not compensate (see, for instance, *WN*, p. 128; pp. 131-2). It is well-known that Smith contrasts projectors with another category of entrepreneurs, "the sober people" (*WN*, p. 357; p. 851), which are said "strangers" to all these "hazardous projects of trade" (*WN*, p. 851) where profit does not compensate the risk of bankruptcy, that is, to unfair and riskier lotteries.

As already noticed by several commentators (D. Levy 1987; S. Hollander 1999; S. Leloup 2000; M. P. Paganelli 2003), the responsibility of the projectors' inclination toward these risky lotteries comes from the tendency to overestimate the chance of gain. It is therefore obvious that their behavior might be viewed as expressing sufficient optimism toward risk in a rank-dependent utility approach (see, for instance, *WN*, p. 128).

But a new difficulty arises: what about sober people? In other words, since we are dealing with so universal tendencies, why are there so few projectors, and so many sober people? Smith is quite clear in chapter 3 of book 2 of the *Wealth of Nations*:

> With regard to misconduct, the number of prudent and successful undertakings is everywhere much greater than that of injudicious and unsuccessful ones. After all our complaints of the frequency of bankruptcies, the unhappy men who fall into this misfortune make but a very small part of the whole number engaged in trade, and all other sorts of business; not much more perhaps than one in a thousand. Bankruptcy is perhaps the greatest and most humiliating calamity which can befall an innocent man. The greater part of men, therefore, are sufficiently careful to avoid it. Some, indeed, do not avoid it; as some do not avoid the gallows. (*WN*, p. 342)

This requires another shift, back from the *Wealth of Nations* to the *Theory of Moral Sentiments*. Smith's already quoted description of sober people's behavior in the *Wealth of Nations*, as "strangers" to all "hazardous projects of trade" (*WN*, p. 851), comes close to what he writes about the virtue of prudence in the *Theory of Moral Sentiments*, which leads us to avoid the possibility of unfavorable events ("any sort of hazard", Smith says; *TMS*, p. 213). Bringing together *sobriety* (in the *Wealth of Nations*) and *prudence* (in the *Theory of Moral Sentiments*) makes visible the moral mechanism which governs the evolution of most entrepreneurs. Smith depicts the long-lasting intimate fight between what he calls our *natural point of view* and the *point of view of the impartial spectator* (see Bréban 2011, chap. 3, 4, 5; 2014). This duality of points of view refers to two alternative perceptions of an individual on his own situation. The natural point of view corresponds to his impulses and leads him to a disproportioned view of his own, whereas the impartial spectator's point of view is a moral reference built on the basis of social interactions and internalised by the individual. And contrary to the natural point of view, the impartial spectator's point of view offers him a proper perspective on his situation. Usually, the latter overcomes the former, thanks to "self-command", which is a part of the virtue of prudence (*TMS*, p. 189). Transposed to the character of the capitalist entrepreneur, this means that although there is a natural tendency to over-value the chances of success, another kind of universality is also at work, so that the impartial spectator's point of view, which properly value the chances of success and failure, supersedes the natural one: the projector progressively puts an end to his over-valuating the probabilities of high outcomes, and becomes a sober man.

Stated more formally, this means that as the projector becomes sober, his decision weights come closer to the probabilities of the corresponding outcomes, till the moment they match them. This, of course, implies a parallel evolution of the function φ of transformation of probabilities: at the end of the process, when the individual is sober enough, there is no more difference between decision weights and probabilities, and φ has become an identity function, which transforms each probability in itself. The behavior of the sober man might therefore be still represented by [22]a and b but, instead of the properties defined by [23], the transformation function is such that

$$\varphi'' = 0 \qquad [24]$$

which clearly implies, along with [22]b, that $\pi_i = p_i$ for each *i*. In such a case, the difference between rank-dependent utility and expected utility vanishes: far from being meaningless with regard to Smith's conception of risk, this latter therefore happens to be the limit case for the sober man, when compared to projectors. Interestingly, though expected utility now appears as a convenient way to approach such behaviour as this of the sober man, it is the result not of some rather demanding rationality assumption (the independence axiom), but of the working of a moral virtue, self-command, of which this demanding rationality is a possible consequence.

4.2. The scope of prudence

Leaving aside, as if it were some kind of black box, the details of the mechanism of self-command which leads the sober man from his natural point of view to the impartial spectator's point of view (see, however, a sketch of a formal representation in Bréban 2011, pp. 139-60), we rather focus on the formal implications of such behaviour from the moment when the impartial spectator's point of view has been adopted. This leaves room to closer investigation on what might be viewed as the certainty component of the behavior of the sober man: his valuation of the intensity of preferences. We already know that $u(x)$ is concave ($u' > 0$ and $u'' \leq 0$; see [19]), in order to express the greater sensitivity to unfavorable events. But carrying on with the bringing together of the sober and the prudent man allows being more specific. Again in the *Theory of Moral Sentiments*, in an already mentioned passage from the first section of part VI, introduced in the 1790 sixth edition, Smith explicitly gives a supplementary content to asymmetric sensitivity to favorable and unfavorable events:

> We suffer more, it has already been observed, when we fall from a better to a worse situation, than we ever enjoy when we rise from a worse to a better. Security, therefore, is the first and the principal object of prudence. It is averse to expose our health, our fortune, our rank, or reputation, to any sort of hazard. It is rather cautious than enterprising, and more anxious: to preserve the advantages which we already possess, than forward to prompt us to the acquisition of still greater advantages. The methods of improving our fortune, which it principally recommends to us, are those which expose to no loss or hazard; real knowledge and skill in our trade or profession, assiduity and industry in the exercise of it, frugality, and even some degree of parsimony, in all our expences. (*TMS*, p. 213)

In these few lines, Smith introduces the principal object of the virtue of prudence, "security", as an effect of our asymmetric sensitivity to favorable and unfavorable events ("We suffer more […] when we fall from a better to a worse situation, than we ever enjoy when we rise from a worse to a better") and contrasts it with "hazard", which refers to the probability of the worst outcomes (an adverse event affecting "our health, our fortune, our rank, or reputation"). That is to say, *prudence* as a moral virtue also expresses a specific kind of aversion: not necessarily toward any kind of risk, but at least toward a risk of loss.

Taking into account that, since his actions are in conformity with the point of view of the impartial spectator, the prudent or sober man behaves like an expected utility decision maker, so that this aversion does not concern decision weights, which could not move away from probabilities. It therefore concerns the function of valuation of preferences $u(x)$ itself. To avoid any confusion, we will refer to it hereafter as "embedded prudence".

Again, this is not an unfamiliar issue. First, because it takes the exact opposite view to, for instance, M. Yaari's dual theory of risk (Yaari 1987), according to which all information concerning attitude toward risk is embedded in the transformation of the probability function – and not in the function of payment which remains linear. Secondly, because arguing, like Smith, that for a prudent man, a risk of loss should be more particularly avoided, echoes the idea that between two reductions in risk with mean-preserving spread, the one concerning losses and the other concerning gains, a prudent man will always prefer the former to the latter: without any reference to Adam Smith, this kind of attitude toward risk has also been called "prudence" by M. Kimball (1990)[1]. Formally, it corresponds to a preference for third

[1] The same notion of "prudence" was previously identified as "downside risk-aversion" by C. Menezes, C. Geiss and J. Tressler (1980). A general framework has been described by L. Eeckhoudt and H. Schlesinger (2006).

degree stochastically dominating lotteries (TSD; see Whitmore 1970[1]) and, in an expected utility framework, it means that the concavity of $u(x)$ is all the more pronounced that the corresponding outcome is low. Or, still in other words, that the third derivative of $u(x)$ is positive. This corresponds to a situation where the simple concavity of $u(x)$ assumed in [19] is not restrictive enough to express embedded prudence, and should be replaced by:

$$u(x) \in U_2^{13} = \{u(x) : u' > 0, u'' \leq 0, u''' \geq 0\} \quad [25]$$

Obviously, [25] combined with an expected utility approach ([22]-[24]) gives an appropriate account of the *prudent* (and, consequently, of the *sober*) man's behavior.

But it might be argued that its extension is still wider and concerns as well the *imprudent* man – here, the *projector*. This does not mean that, leaving risk aside, an individual, either prudent or imprudent, might be represented in the same way (Bréban 2011, pp. 273-88) but that, leaving aside with risk all other differentiating aspects like intertemporal preferences or the estimation of the consequences of actions, there is no reason why it should be represented differently. Since security is, for Smith, a consequence of asymmetric sensitivity which grants a higher impact to unfavorable events (*TMS*, p. 213), and since this type of asymmetry depends on the place of the individual's ordinary happiness within a social scale which goes from the "lowest depth of misery" to the "highest pitch of human prosperity" (*TMS*, p. 45; see Bréban 2012), it does not depend on his attitude toward risk (for instance, projector or sober man), but only on its level of happiness. This already provides reasons to argue that a function with embedded prudence [25] represents the underlying function of intensity of preference of both the prudent and the imprudent man, the difference between them lying in the fact that, though a rank-dependent utility approach can give an appropriate account of both behaviours, this approach vanishes into the particular case of expected utility for the former ([22]-[24]), and of (non-expected) rank-dependent utility for the latter ([22]-[23]).

But we also have other reasons. The example of the lottery of the army, compared to that of the sea, is of special interest. At the difference from the case of the lottery of the law compared to the lottery of common trades, the worst outcome (remaining a private or a sailor) corresponds to a better situation in the former lottery, which is also the preferred one. It has been shown (*supra*, pp. 11 *sqq*) that such a preference might be explained by third-degree inverse stochastic dominance of the second type (TISD2). In a rank-dependent utility framework, it can be argued that this property depends not only on the function of transformation of probabilities $\varphi(p)$ itself, but on the properties of $u(x)$ which, although

[1] TSD is formally characterized as follows. Let $F(x)$ and $G(x)$ be two different functions of cumulative probabilities defined on an interval [a, b], $H_1(x) = F(x) - G(x)$, $H_2(x) = \int_a^x H_1(t)dt$. Then, $\int_a^x H_2(t)dt \leq 0$ is equivalent to F TSD G.

increasing concave, also exhibits a non-negative third derivative, that is, embedded prudence like in [25]. On an appropriate subset of the pair of lotteries ordered by TISD2, the concavity of $\varphi(p)$ might therefore produce a risk-seeking attitude by overcoming the concavity of $u(x)$. However, it might also respect the prudence involved in TISD2, so that risk-seeking appears as increasing with the outcome. On the contrary, in the case of the comparison of the lotteries of the law and of common trades, SISD supposes that the properties of $\varphi(p)$ have also superseded the non-negativity of u'''.

If this interpretation is accepted, it would mean that most individuals (actually, all those whose ordinary state of happiness is high enough) are potentially prudent, because of the characteristics of their function of evaluation of preferences, already displaying embedded prudence [25]. However, when they keep a *natural* point of view on their own situation, this potential prudence toward risk might be, in a context of risk-seeking, either hidden for some people who prefer dominating lotteries in the sense of SISD (lottery of the law, preferred to the lottery of the common trades), or disclosed, for all those who prefer dominating lotteries in the sense of TISD2 (lottery of the army, preferred to the lottery of the sea)[1]. But when they reach the point of view of the *impartial spectator*, the possibly contradictory action of $\varphi(p)$ is suppressed, so that embedded prudence involved in [25] always gives birth to prudence toward risk in the ranking of lotteries.

5. CONCLUDING REMARKS

The pages on lotteries in the *Wealth of Nations*, supplemented by the passages from the *Theory of Moral Sentiments* on prudence, give rise to some non-trivial propositions:

1. Preferences between lotteries can be expressed through a functional $RDU(L) = \sum_{i=1}^{n} \pi_i u(x_i)$ (see [22]) which combines
    - a certainty component, $u(x_i)$ which reflects the intensity of preferences on possible outcomes;
    - a risk component π_i which, when different from probabilities p_i, shows how probabilities are transformed into decision weights.
2. The properties of the certainty component $u(x)$ depends on the position of the individual concerned in a social scale of happiness. For most individuals, it implies that $u(x)$ is concave (greater effect of an unfavorable event; Bréban 2012), with a non-negative third derivative (embedded prudence, see [25]).

[1] In another framework, D. Crainich, L. Eeckhoudt and A. Trannoy (2011) supported the idea that a certain kind of risk-seeking (*mixed* risk-seeking) is consistent with prudence.

3. The decision weights π_i (risk component) depend on a transformation of probabilities according to their ranks: $\pi_i = \varphi\left(\sum_{j=i}^{n} p_j\right) - \varphi\left(\sum_{j=i+1}^{n} p_j\right)$, with $\varphi(0) = 0$ and $\varphi(1) = 1$. The assumed concavity of φ (optimism toward risk, in rank-dependent utility models) is an expression of the over-valuation of the chances of success and of the under-valuation of the chances of failure (see [23]), which Smith links to the balance between fear and hope.
4. The point of view of an individual on his own situation goes from the natural (imprudence) to the impartial spectator's (prudence) point of view (Bréban 2014), and determines his ranking of lotteries through the subsequent modifications of the risk component – and not of the certainty component. Attitudes toward risk of both typical points of view are characterized as follows:
   - Natural point of view (imprudent individual; see [22], [25], [23])

     $u(x): u' > 0, u'' \leq 0, u''' \geq 0$

     $\varphi' > 0, \varphi'' < 0$

     This allows for the risk-seeking attitude, in the sense of preference given to dominating lotteries according to a subset of SISD ([9]) or TISD2 (where risk-seeking comes along with prudence toward risk; [15]), that Smith describes and illustrates in the pages on lotteries from the *Wealth of Nations*.
   - Impartial spectator's point of view (prudent individual; see [22], [25], [24])

     $u(x): u' > 0, u'' \leq 0, u''' \geq 0$

     $\varphi' > 0, \varphi'' = 0$

     This prudent behavior can be equivalently described either as a limit-case of rank dependent utility with $\varphi'' = 0$, or through a standard expected utility approach with $\pi_i = p_i$, so that the properties of the utility function incorporate both prudence ($u''' \geq 0$) and adversity toward risk ($u'' \leq 0$), such as found in some positive figures of economic agents in the *Wealth of Nations*.
5. An intra-individual transformation, based on "self-command", from the natural point of view into the impartial spectator's point of view (Bréban 2014), is successfully carried out for some individuals, and gives rise, along with prudence, to the progressive prevalence of aversion toward risk. This is achieved in spite of the universal tendency which leads to over-value probabilities of success and under-value probabilities of failure.

These propositions perform a sophisticated juncture between elements rooted in Adam Smith's economic and moral works, in which we can acknowledge a complete theory of behavior under risk.

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